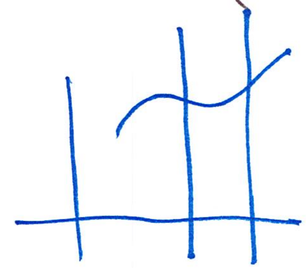
Chapter 14 Partial Derivatives (10 lectures)



§14.1 Functions of Several Variables

- functions of two variables

$$z = f(x, y)$$

z: dep. var.; x, y: indep. variables

domain $D = \{(x,y) \mid f(x,y) \text{ is well defined}\}$

range $R = \{f(x,y) \mid (x,y) \in D\}$

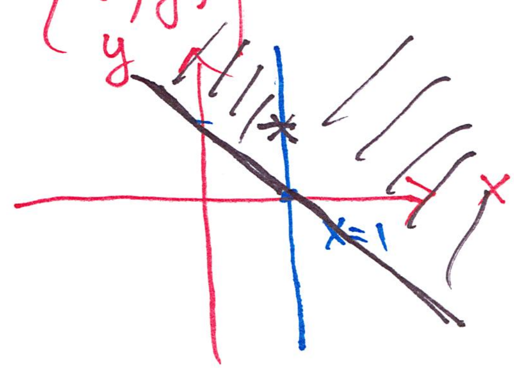
$\forall (x,y) \in D$, $f(x,y)$ has a single value

Ex. 1 (a) $f(x,y) = \dfrac{\sqrt{x+y-1}}{x-1}$; (b) $f(x,y) = x \ln(y^2 - x)$; (c) $f(x,y) = \dfrac{\sqrt{y-x^2}}{1-x^2}$

evaluate $f(3,2)$, find and sketch the domain

(a) $\begin{cases} x-1 \neq 0 \Rightarrow x \neq 1 \\ x+y-1 \geq 0 \Rightarrow x+y \geq 1 \end{cases}$

$D = \{(x,y) \mid x \neq 1 \text{ and } x+y \geq 1\}$

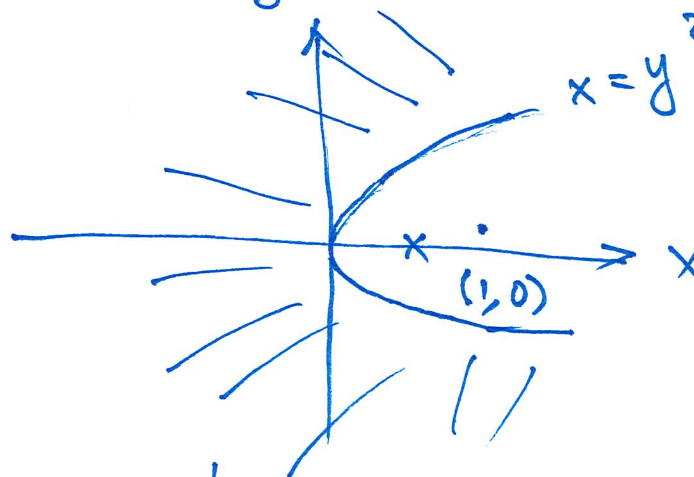


$x+y=1$

(b) $f(x,y) = x \ln(y^2 - x)$ $\quad D = \{(x,y) \mid y^2 > x\}$

$y^2 - x > 0 \qquad y^2 > x$

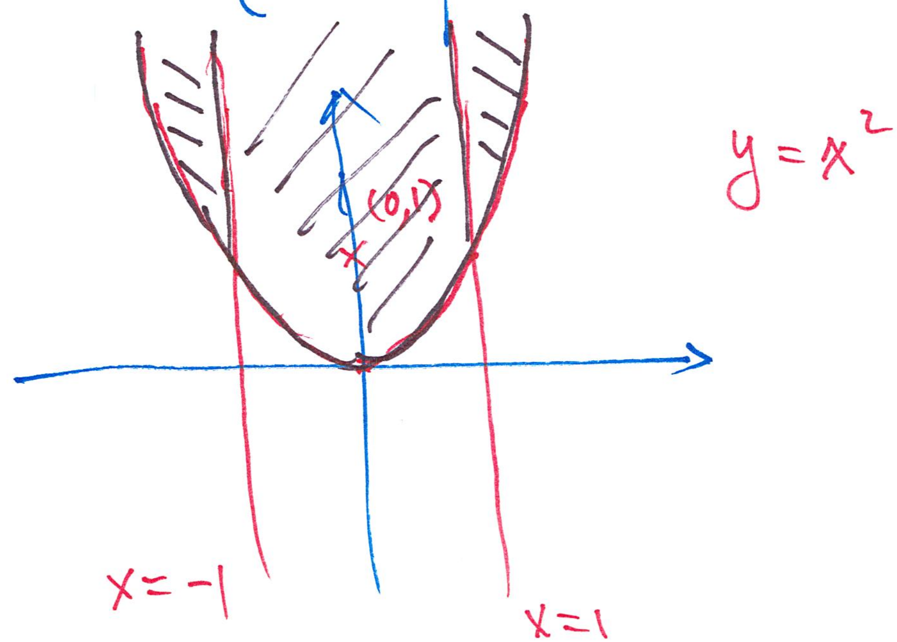
$y^2 = x$



(c) $f(x,y) = \dfrac{\sqrt{y - x^2}}{1 - x^2}$

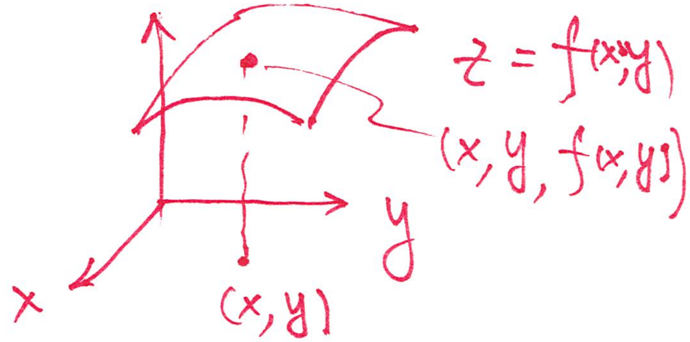
$D = \{(x,y) \mid x \neq \pm 1 \text{ and } y \geq x^2\}$

$$\begin{cases} 1 - x^2 \neq 0 \\ y - x^2 \geq 0 \end{cases} \Rightarrow \begin{cases} x \neq \pm 1 \\ \underline{y \geq x^2} \end{cases}$$



- **graphes of $z = f(x,y)$**

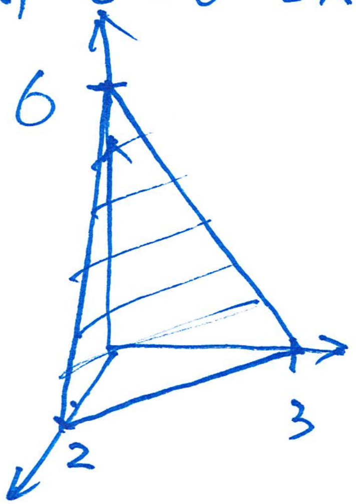
$$S = \{ (x, y, f(x,y)) \mid (x,y) \in D \}$$



examples sketch the graphes of

(a) $f(x,y) = 6 - 3x - 2y$; (b) $g(x,y) = \sqrt{9 - x^2 - y^2}$; (c) $h(x,y) = 4x^2 + y^2$

(a) $z = 6 - 3x - 2y$



$x + y + z = 0$

$0 = 6 - 3x - 0$

$0 = 6 - 0 - 2y$

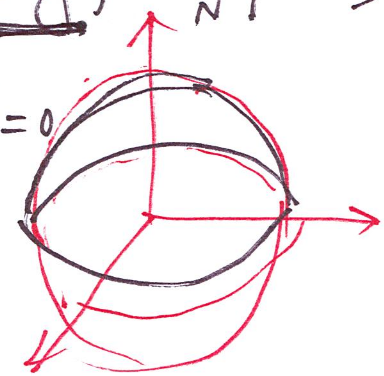
$z = 6 - 0 - 0$

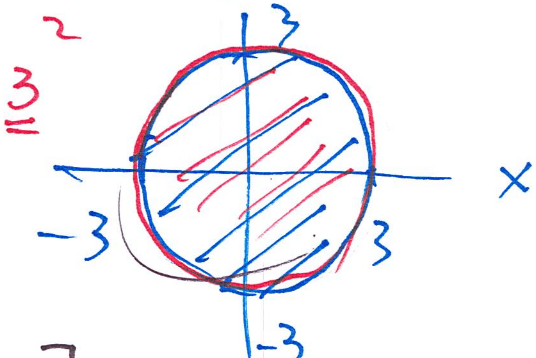
(b) $z = \sqrt{9 - x^2 - y^2}$ $\quad z^2 = 9 - x^2 - y^2 \geq 0.$

$z^2 = 9 - x^2 - y^2$ $\quad \Rightarrow 9 \geq x^2 + y^2$

$z = \sqrt{9 - (x^2 + y^2)} \leq \sqrt{9} = 3$ $\quad x^2 + y^2 + z^2 = 3^2$

$\geq \sqrt{9 - 9} = 0$





$R = [0, 3]$

- <u>level curves</u>

$$C = \{ (x,y) \mid f(x,y) = k \} \qquad k\text{ — constant}$$

<u>examples</u> sketch level curves of

(a) $f(x,y) = 6 - 3x - 2y$ for $k = -6, 0, 6, 12$; (b) $g(x,y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$

(c) $h(x,y) = 4x^2 + y^2 + 1$ for $k = 2, 3$.

(a) $6 - 3x - 2y = -6 = k$

$\underline{3x + 2y = 12}$

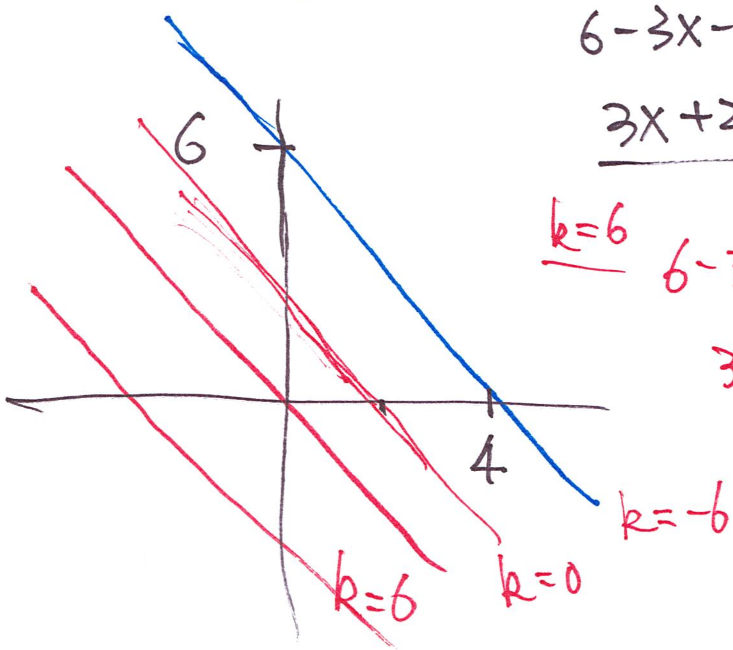
$\underline{k=0}$

$6 - 3x - 2y = 0$

$\underline{3x + 2y = 6}$

$\underline{k=6}$ $6 - 3x - 2y = 6$

$3x + 2y = 0$

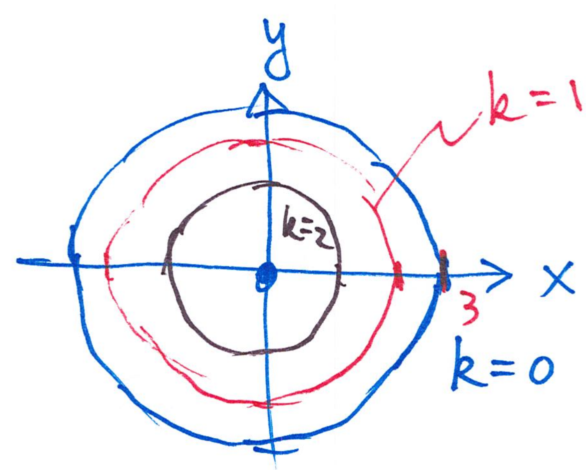


$z = \sqrt{9 - x^2 - y^2} = 0 \Longrightarrow \underline{9 = x^2 + y^2}$

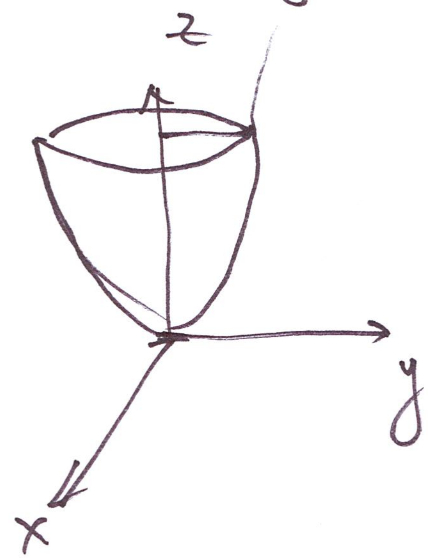
$\underline{k=1}$ $\sqrt{9 - x^2 - y^2} = 1 \Longrightarrow x^2 + y^2 = 8$

$\underline{k=3}$ $\sqrt{9 - x^2 - y^2} = 3 \Longrightarrow x^2 + y^2 = 0$

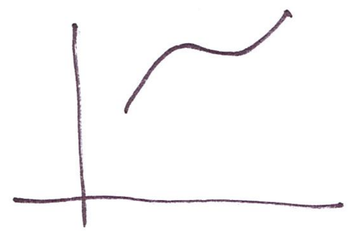
$\underline{k=2}$ $\sqrt{9 - x^2 - y^2} = 2 \Longrightarrow x^2 + y^2 = 5$



(c) $z = h(x,y) = 4x^2 + y^2$ $\quad D = R^2$

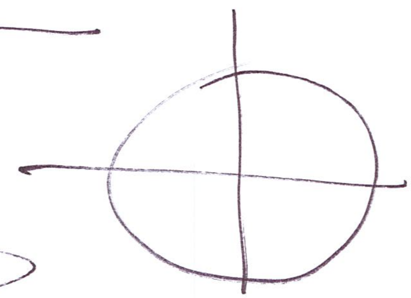


$R = [0, +\infty)$

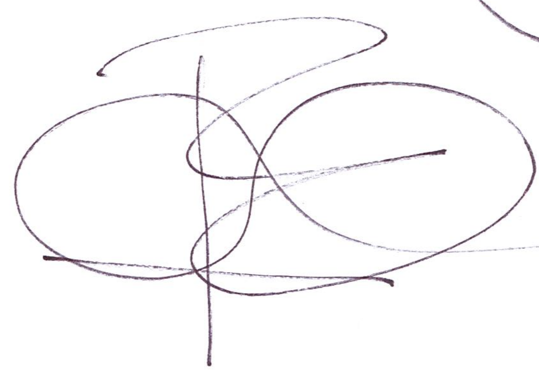


① graph $\quad y = f(x)$

② level curve $\quad f(x,y) = k$



③ parametric equation $\quad \begin{cases} x = f(t) \\ y = g(t) \end{cases}$



- Functions of three variables

$$w = f(x, y, z)$$

domain $D = \{(x,y,z) \mid f \text{ is well-defined}\}$

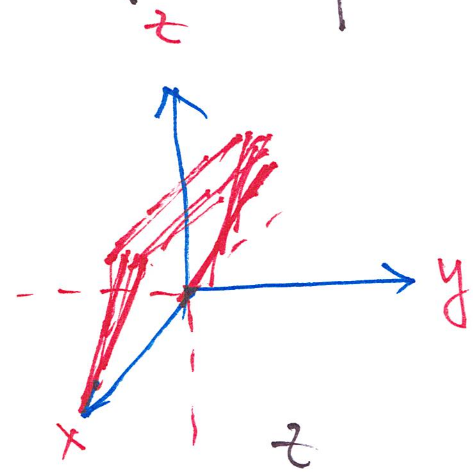
graph $\Omega = \{(x,y,z,f(x,y,z)) \mid (x,y,z) \in D\}$

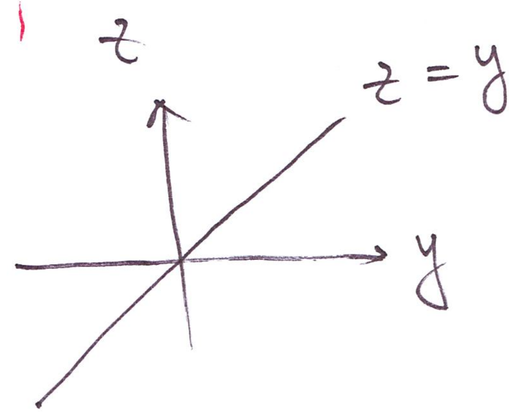
level surface $S = \{(x,y,z) \mid f(x,y,z) = k\}$

ex. 14 $f(x,y,z) = \ln(z-y) + xy\sin z$, $D = ?$

$z - y > 0$ $z > y$ $z = y$

$D = \{(x,y,z) \mid z > y\}$





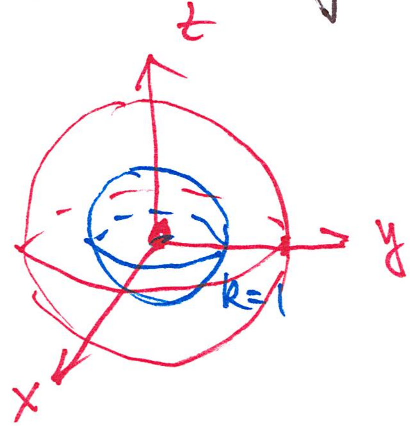
ex. 15 $f(x,y,z) = x^2 + y^2 + z^2$, find the level surfaces.

$x^2 + y^2 + z^2 = k$

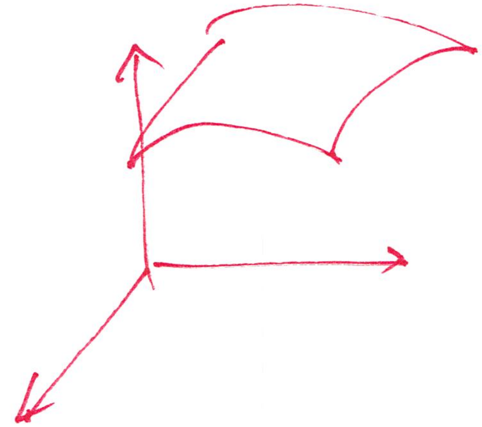
$k=0$ $x^2 + y^2 + z^2 = 0$

$k=1$ $x^2 + y^2 + z^2 = 1$

$k=4$ $x^2 + y^2 + z^2 = 2^2$

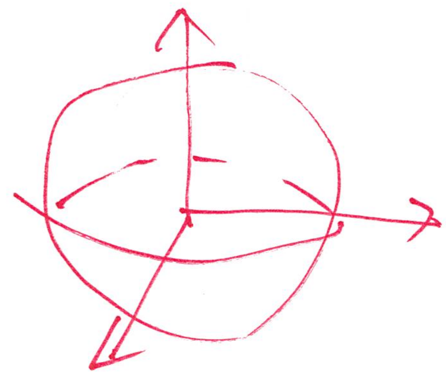


(1) graph $z = f(x, y)$



(2) level surface

$$f(x, y, z) = k$$



(3) parametric representation