Review for Exam 1

Chapter 12 $\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$ | $\vec{a}\cdot\vec{b}=0 \Leftrightarrow \vec{a}\perp\vec{b}$

$\vec{a}\times\vec{b}$, $|\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}|\sin\theta$ | $\vec{a}\times\vec{b}=\vec{0} \Leftrightarrow \vec{a}\,/\!/\,\vec{b}$

equations
- line $\langle x-x_0, y-y_0, z-z_0\rangle = t\langle a,b,c\rangle$
- plane $\langle x-x_0, y-y_0, z-z_0\rangle\cdot\vec{n} = 0$

quadratic surfaces (P830, table 1)

Chapter 13 $\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$, $\vec{v}(t) = \vec{r}\,'(t)$, $\vec{a}(t) = \vec{v}\,'(t)$

arc length $L = \int_a^b |\vec{r}\,'(t)|\,dt$, $s = s(t) = \int_a^t |\vec{r}\,'(u)|\,du$

curvature $\kappa = \left|\dfrac{d\vec{T}}{ds}\right| = \dfrac{|\vec{T}'(t)|}{|\vec{r}\,'(t)|}$, $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

normal vector $\vec{N} = \dfrac{\vec{T}'(t)}{|\vec{T}'(t)|}$

Chapter 14

- limit $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ DNE if limits along 2 paths are different
- continuity $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$
- partial derivatives
- chain rule $D f(\vec{g}(\vec{x})) = D f(\vec{g}(\vec{x}))\, D\vec{g}(\vec{x})$
- directional derivative $D_{\vec{u}} f(P) = \nabla f(P) \cdot \vec{u}$ where $|\vec{u}| = 1$.
- gradient vector $\begin{cases} \nabla f \text{ — the direction in which } f \text{ increases most rapidly} \\ \nabla f \perp \{(x,y,z) \mid f(x,y,z) = k\} \end{cases}$

  the tangent plane eq. $\nabla f(P) \cdot \langle x-x_0, y-y_0, z-z_0 \rangle = 0$

  the normal line eq. $\langle x-x_0, y-y_0, z-z_0 \rangle = t\, \nabla f(P)$
- linear approximation $f(x,y) \approx f(x_0,y_0) + \nabla f(x_0,y_0) \cdot \langle x-x_0, y-y_0 \rangle$
- differential $\Delta f(x_0,y_0) = f(x_0+\Delta x, y_0+\Delta y) - f(x_0,y_0)$

  $\approx df = \nabla f(x_0,y_0) \cdot \langle \Delta x, \Delta y \rangle$

- Extreme values

critical pt $P=(x_0,y_0)$: $\nabla f(P) = \langle 0,0\rangle$ or DNE

2^{nd} der. test : $D(P) = \left(f_{xx}f_{yy} - f_{xy}^2\right)(P)$

$$\begin{cases} D(P)>0 & \begin{cases} f_{xx}(P)<0 \Longrightarrow f(P) - \text{l. max} \\ f_{xx}(P)>0 \Longrightarrow f(P) - \text{l. min} \end{cases} \\ D(P)<0 \Longrightarrow \text{saddle pt.} & \end{cases}$$

- Lagrange multiplier

$$\max_{g(x,y)=c}/\min f(x,y)$$

critical pts : $\begin{cases} \nabla f = \lambda \nabla g \\ g = c \end{cases}$