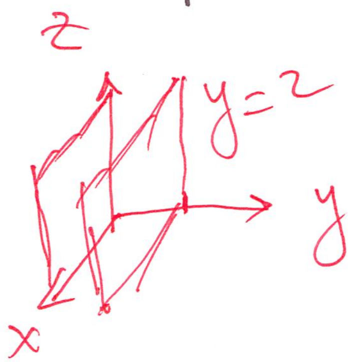
Lesson 1

(1) Equation of a simple plane

§13.2 #20 (P824) Sketch the plane parallel to the xz-plane containing the pt (1, 2, 3).

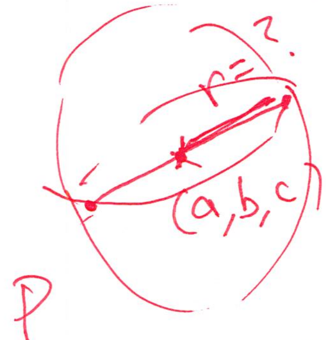


$y = 2$

$(a+b)^2$
$= a^2 + b^2 + 2ab$
$a^2 - b^2 = (a+b)(a-b)$

(2) Equation of a sphere/ball

§13.2 #28 Find an equation of the sphere passing through P(−4, 2, 3) and Q(0, 2, 7) with its center at the midpt of PQ



$(a, b, c) = \left(\frac{-4+0}{2}, \frac{2+2}{2}, \frac{3+7}{2} \right)$
$= (-2, 2, 5)$

$r = \sqrt{(-4+2)^2 + (2-2)^2 + (3-5)^2}$

Give a geometric description of the following sets of pts

#31. $x^2 + y^2 + z^2 - 2y - 4z - 4 = 0$

$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

$0 = x^2 + (y-1)^2 - 1 + (z-2)^2 - 4 - 4$

$x^2 + (y-1)^2 + (z-2)^2 = 3^2$

#34. $x^2 + y^2 - 14y + z^2 \le -13$

$2 \cdot 7 \cdot y + 7^2 - 7^2$

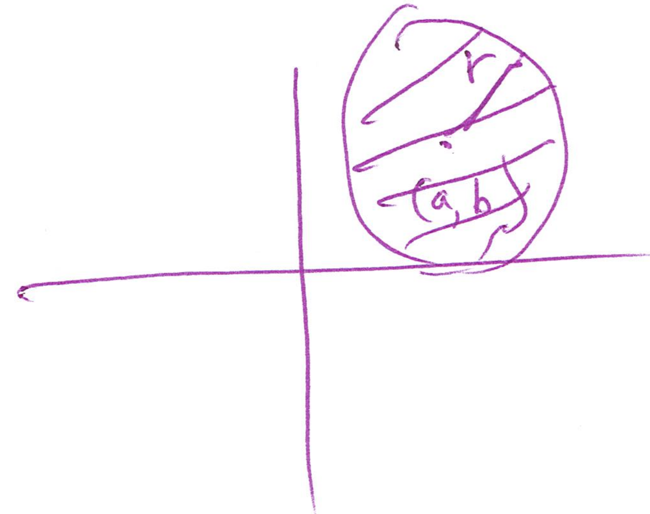
$x^2 + (x-7)^2 + z^2 \le 7^2 - 13$

#75(a). Determine whether the pts P, Q, and R are collinear (lie on a line) by comparing $\vec{PQ}$ and $\vec{PR}$

(a) P(1, 6, −5), Q(2, 5, −3), R(4, 3, 1)

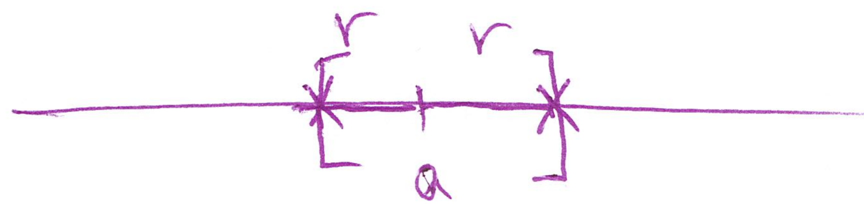
$$(x-a)^2 + (y-b)^2 = r^2$$

$$\leq r^2$$



$$(x-a)^2 = r^2$$

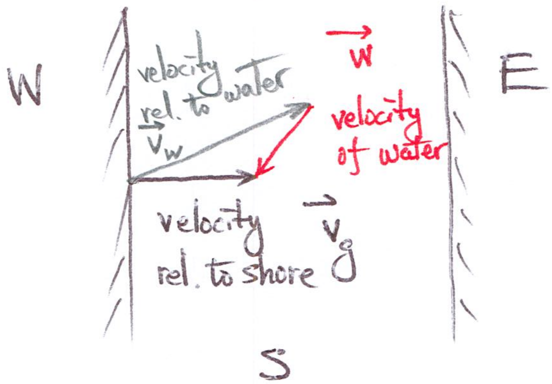
$$\leq r$$



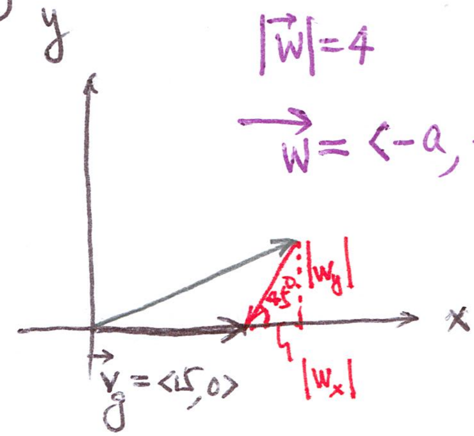
Vectors

Example 6 (§13.1, p811) Speed of a boat in a current

Suppose the water in a river moves south-west (45° west of south) at 4 mi/hr and a motorboat travels due east at 15 mi/hr relative to the shore. Determine the speed of the boat and its heading relative to the moving water.



$$\vec{V}_g = \vec{V}_w + \vec{W}$$



$$\left.\begin{array}{l} |\vec{W}| = 4 \\ \vec{W} = \langle -a, -a \rangle \end{array}\right\} \Rightarrow \vec{W}$$

$\vec{W} = -2\sqrt{2}\langle 1, 1$

Given $|\vec{W}| = 4$

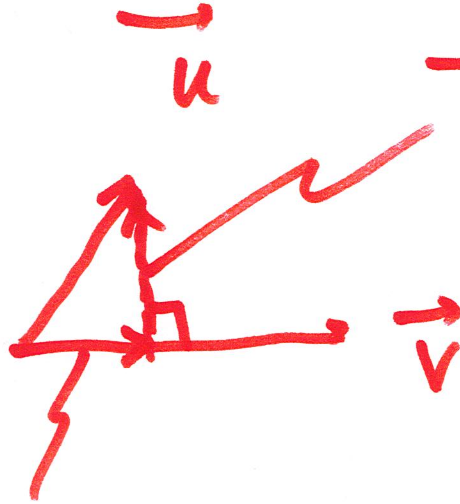
$\vec{V}_g = \langle 15, 0 \rangle$

Find $|\vec{V}_w| = ?$

$$\vec{V}_w = \vec{V}_g - \vec{W} = \langle 15, 0 \rangle + 2\sqrt{2}\langle 1, 1 \rangle$$

$$= \langle 15 + 2\sqrt{2}, 2\sqrt{2} \rangle$$

$$|\vec{V}_w| = \sqrt{(15+2\sqrt{2})^2 + (2\sqrt{2})^2} \approx 18$$



$$\vec{u} - \mathrm{proj}_{\vec{v}}\vec{u} = \vec{u} - \alpha\vec{v}$$

$$? = \mathrm{proj}_{\vec{v}}\vec{u} = \alpha\vec{v} \qquad \alpha = ?$$

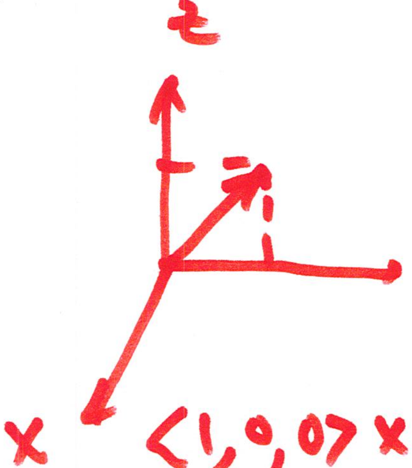
$$0 = (\vec{u} - \alpha\vec{v})\cdot\vec{v}$$

$$= \vec{u}\cdot\vec{v} - \alpha\,\vec{v}\cdot\vec{v} \Longrightarrow \alpha = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|\,|\vec{v}|}$$

$$\mathrm{proj}_{\vec{v}}\vec{u} = \left[\vec{u}\cdot\left(\frac{\vec{v}}{|\vec{v}|}\right)\right]\frac{\vec{v}}{|\vec{v}|}$$

§13.3, #54 (P835)

Find two vectors that are orthogonal to $\langle 0,1,1\rangle$ and to each other.



$\underline{\langle 1,0,0\rangle}$

$0=\langle u_1,u_2,u_3\rangle\cdot\langle 0,1,1\rangle$

$= u_2+u_3$

$\langle 1,0,0\rangle \times \langle 0,1,1\rangle$

§13.4, #60 (P844) <u>Vector equation</u>

Find all vectors $\vec{u}$ that satisfy

$$\langle 1,1,1\rangle \times \vec{u} = \langle 0,0,1\rangle$$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ u_1 & u_2 & u_3 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ u_2 & u_3 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 1 \\ u_1 & u_3 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 1 \\ u_1 & u_2 \end{vmatrix}\vec{k}$$

$$= \langle u_3-u_2,\ u_1-u_3,\ u_2-u_1\rangle = \langle 0,0,1\rangle$$

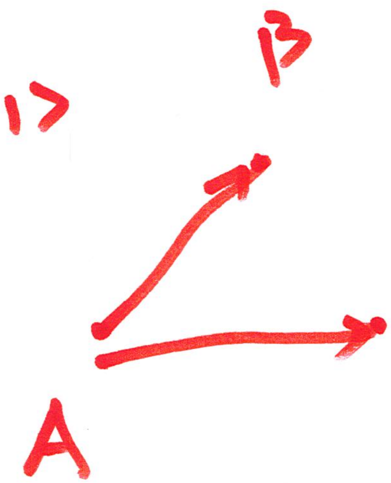
$$\left\{\begin{array}{l} \left.\begin{array}{l} u_3-u_2=0 \\ u_1-u_3=0 \end{array}\right\}\ u_1-u_2=0 \implies u_1=u_2 \\ u_2-u_1=1 \implies 0=1 \ \text{contradition} \end{array}\right.$$

<u>no vector</u>

§13.4, #33 (P843)

Find the area of the triangle T with verti

A(0,0,0), B(3,0,1), and C(1,1,0)



$$\frac{|\vec{AB}\times\vec{AC}|}{2}$$