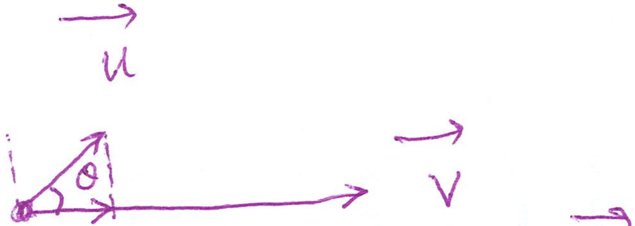
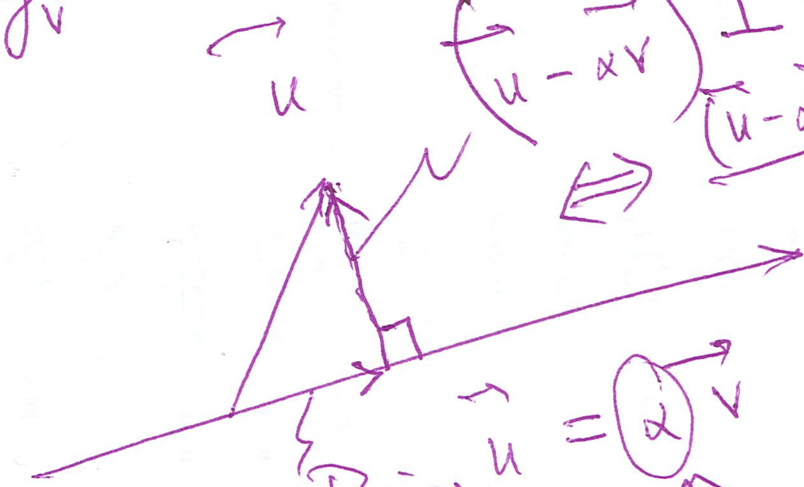
$$\frac{\vec u \cdot \vec v}{|\vec v|} = \frac{|\vec u|\,\cancel{|\vec v|}\cos\theta}{\cancel{|\vec v|}}$$



$$|\vec u|\cos\theta\,\frac{\vec v}{|\vec v|} = \mathrm{proj}_{\vec v}\vec u$$

$$\left(\vec u \cdot \frac{\vec v}{|\vec v|}\right)\frac{\vec v}{|\vec v|}$$

$\vec u$



$$(\vec u - \alpha\vec v) \perp \vec v \iff (\vec u - \alpha\vec v)\cdot\vec v = 0$$

$$\mathrm{proj}_{\vec v}\vec u = \alpha\vec v$$

$$\alpha = \frac{\vec u \cdot \vec v}{\vec v \cdot \vec v}$$

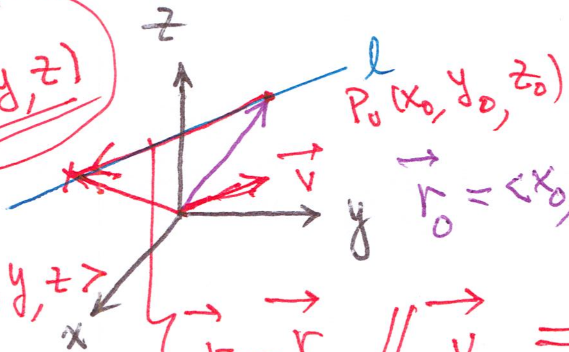
Lesson 2

Equation of a line l

Given: two pts $P_0(x_0, y_0, z_0)$ and $P_1(x_1, y_1, z_1)$ on l

or: a pt $P_0(x_0, y_0, z_0)$ on l
and a vector $\vec{v} = \langle a, b, c \rangle \parallel l$

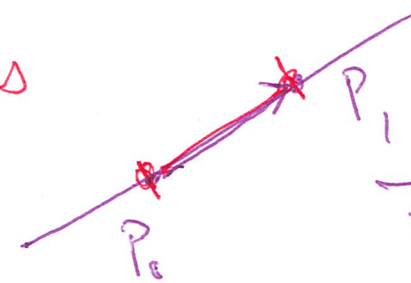
$P(x, y, z)$



$\vec{r} = \langle x, y, z \rangle$ $\quad \vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

$\vec{r} - \vec{r}_0 \parallel \vec{v} \implies \vec{r} - \vec{r}_0 = t\vec{v} \implies \vec{r} = \vec{r}_0 + t\vec{v}$ vector eq.

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$ parametric equations



$\vec{v} = \overrightarrow{P_0P_1} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$

Find equation of lines

#12. ~~the~~ line through $(0,0,1)$ in the direction $\vec{v} = \langle 1, -2, 0 \rangle$

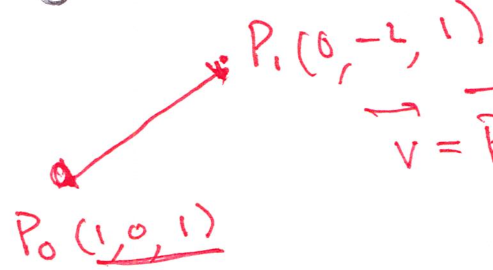
$$\begin{cases} x = 0 + t \\ y = 0 - 2t \\ z = 1 \end{cases}$$

#20. the line through $(-3, 4, 2)$ that is perpendicular to both $\vec{u} = \langle 1, 1, -5 \rangle$ and $\vec{v} = \langle 0, 4, 0 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 1 & -5 \\ 0 & 4 & 0 \end{vmatrix} = \langle 20, 0, 4 \rangle = 4\langle 5, 0, 1 \rangle$$

$$\begin{cases} x = -3 + 5t \\ y = 4 \\ z = 2 + t \end{cases}$$

Line segments #28 Find parametric equations for the line segment joining (1,0,1) and (0,-2,1)



$\vec{v} = \overrightarrow{P_0P_1} = \langle 0, -2, 1\rangle - \langle 1, 0, 1\rangle = \langle -1, -2, 0\rangle$

$$\begin{cases} x = 1 - t \\ y = \quad -2t \\ z = 1 \end{cases} \qquad \boxed{0 \le t \le 1}$$

Parallel, intersecting, or skew lines Determine whether the following pairs of lines are parallel, interset at a single pt, or are skew. **Parallel**, determine whether they are the same line. **Interset**, determine the point of intersection. **Colliding particles**?

$\vec{u} \parallel \vec{v} \iff \exists\, \alpha \neq 0$, s.t. $\vec{u} = \alpha \vec{v}$

#34 $\begin{cases} x = 4 + 5t \\ y = \quad -2t \\ z = 1 + 3t \end{cases}$ and $\begin{cases} x = 10s \\ y = 6 + 4s \\ z = 4 + 6s \end{cases}$

$\vec{u} = \langle 5, -2, 3\rangle \neq \alpha \vec{v} = \langle 10, 4, 6\rangle \Rightarrow$ not parallel

$\boxed{t = s}$

intersection

$$\left.\begin{aligned} 4 + 5t &= 10s \\ -2t &= 6 + 4s \\ 1 + 3t &= 4 + 6s \end{aligned}\right\} \Rightarrow t, s$$

$\begin{cases} 10s - 5t = 4 \\ 4s + 2t = -6 \end{cases}$

$-5t - \frac{5}{2} \cdot 2t = 4 + \frac{5}{2} 6$

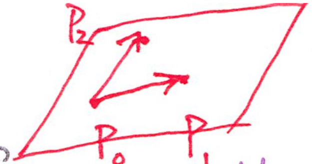
$-10t = 19 \Rightarrow t = -\frac{19}{10}$, $s = \frac{1}{10}\left[4 + 5 \cdot \left(-\frac{19}{10}\right)\right] = \frac{1}{10}\left[\frac{8}{2} - \frac{19}{2}\right] = -\frac{11}{20}$

$1 + 3 \cdot \left(-\frac{19}{10}\right) = \frac{10 - 57}{10} = \frac{-47}{10}$

$4 + 6s = 4 + 6 \cdot \frac{-11}{20} = \frac{40 - 33}{10} = \frac{7}{10} \neq -\frac{47}{10}$

skew lines

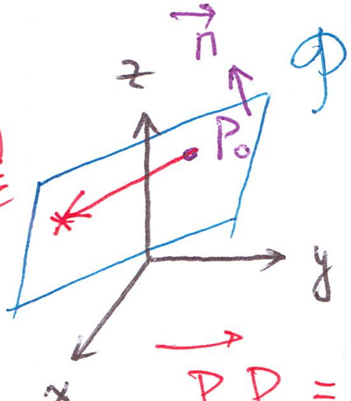
Equation of a Plane in R^3 $\mathcal{P}$



Given: • three pts $P_i(x_i, y_i, z_i)$, $i = 0, 1, 2$ → $\vec{n} = \vec{P_0P_1} \times \vec{P_0P_2}$

or • $\left[\begin{array}{l} \text{a pt } P_0(x_0, y_0, z_0) \text{ on } \mathcal{P} \\ \text{a vector } \vec{n} = \langle a, b, c\rangle \perp \mathcal{P} \end{array}\right.$

$P(x, y, z)$



$\vec{P_0P} = \langle x - x_0, y - y_0, z - z_0 \rangle \perp \vec{n}$

$ax + by + cz = ax_0 + by_0 + cz_0$

$0 = \langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle$

$= a(x - x_0) + b(y - y_0) + c(z - z_0)$

#46 The plane that is parallel to the vectors $\langle 1, -3, 1\rangle$ and $\langle 4, 2, 0\rangle$ passing through the pt $(3, 0, -2)$.

$\vec{n} = \langle 1, -3, 1\rangle \times \langle 4, 2, 0\rangle$

$\vec{n}_1 = \langle 2, 2, -3\rangle$, $\vec{n}_2 = \langle -10, -10, 15\rangle = -5\langle 2, 2, -3\rangle$

#66 $\begin{cases} 2x + 2y - 3z = 10 \\ -10x - 10y + 15z = 10 \end{cases}$ parallel, orthogonal, or neither?

$\vec{n}_1 \cdot \vec{n}_2 = -18 - 20 - 3 \neq 0$

#67 $\begin{cases} 3x + 2y - 3z = 10 \\ -6x - 10y + z = 10 \end{cases}$ $\vec{n}_1 = \langle 3, 2, -3\rangle$, $\vec{n}_2 = \langle -6, -10, 1\rangle$ not parallel

#69 Q: $3x-2y+z=12$

R: $-x+\frac{2}{3}y-\frac{1}{3}z=0$

S: $-x+2y+7z=1$

T: $\frac{3}{2}x-y+\frac{1}{2}z=6$

pairs of planes: parallel, orthogonal, or identical?

$\vec{n}_Q=\langle 3,-2,1\rangle$

$\vec{n}_R=\langle -1,\frac{2}{3},-\frac{1}{3}\rangle=\frac{1}{3}\langle -3,2,-1\rangle$ Q // R

$\vec{n}_S=\langle -1,2,7\rangle$

$\vec{n}_T=\langle \frac{3}{2},-1,\frac{1}{2}\rangle=\frac{1}{2}\langle 3,-2,1\rangle$ Q // T // R

#74 Q: $x+2y-z=1$

R: $x+y+z=1$

Find an equation of the line of intersection of the planes Q and R

a pt and a vector

a pt

$z=0$

$\begin{cases} x+2y=1 \\ x+y=1 \end{cases} \Rightarrow \begin{matrix} y=0 \\ x=1 \end{matrix}$

a pt $(1,0,0)$

a vector

$\vec{v}=\vec{n}_1\times\vec{n}_2=\langle 1,2,-1\rangle\times\langle 1,1,1\rangle=\begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$=\langle 3,-2,-1\rangle$

$\begin{cases} x=1+3t \\ y=-2t \\ z=-t \end{cases}$

#79 plane: $3x+2y-4z=-3$

Find the intersection pt of the plane and the line.

line $\begin{cases} x=-2t+5 \\ y=3t-5 \\ z=4t-6 \end{cases}$

$-3=3\cdot[-2t+5]+2\cdot[3t-5]-4[4t-6]$

$=[-6t+15]+[6t-10]-[16t-24]$

$=-16t+29 \Rightarrow t=\frac{1}{16}[29+3]=2$

$(-2t+5,3t-5,4t-6)\big|_{t=2}=(1,1,2)$