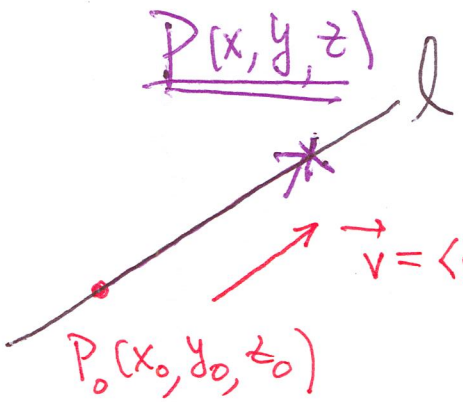
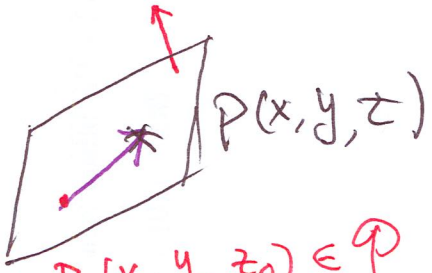


$$\overrightarrow{P_0P} = \langle x - x_0,\ y - y_0,\ z - z_0 \rangle \text{ on } l$$

$$\Rightarrow \overrightarrow{P_0P} \parallel \vec{v} \qquad \overrightarrow{P_0P} \parallel \vec{r} - \vec{r}_0$$

$$\overrightarrow{P_0P} = t\,\vec{v} \quad \text{param.}$$

$$\begin{cases} x - x_0 = at \\ y - y_0 = bt \\ z - z_0 = ct \end{cases}$$

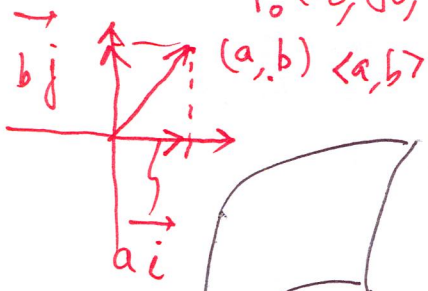


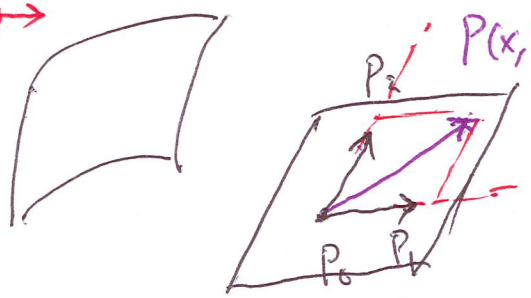
$$\overrightarrow{P_0P} = \langle x - x_0,\ y - y_0,\ z - z_0 \rangle \text{ on } \varphi$$

$$\Rightarrow \vec{n} \perp \overrightarrow{P_0P}$$

$$\Rightarrow 0 = \vec{n} \cdot \overrightarrow{P_0P} = \langle a, b, c \rangle \cdot \langle x - x_0,\ y - y_0,\ z - z_0 \rangle$$

$$= a(x - x_0) + b(y - y_0) + c(z - z_0)$$





$$\overrightarrow{P_0P} = \alpha\,\overrightarrow{P_0P_1} + \beta\,\overrightarrow{P_0P_2}$$

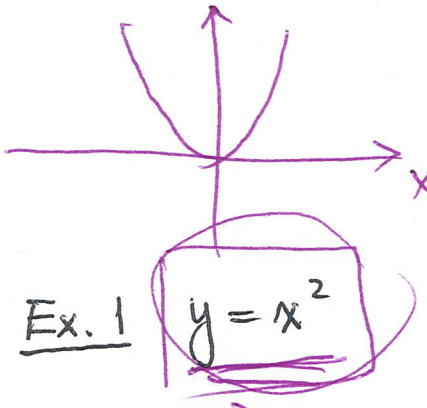
para.

Lesson 3

§13.6 Cylinders and Quadric Surfaces

Cylinder: a surface that consists of all lines that are parallel to a given line and pass through a given curve.

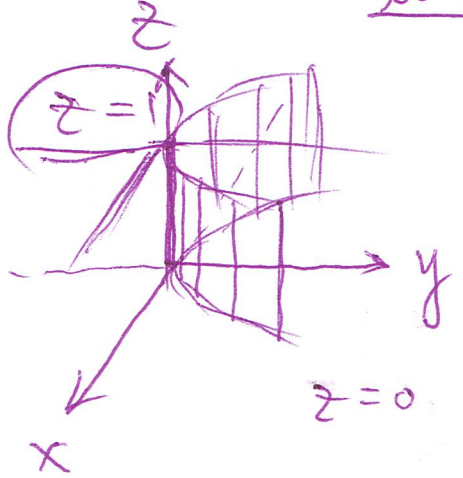
Ex. 1 $y = x^2$



xy-plane $z = 0$

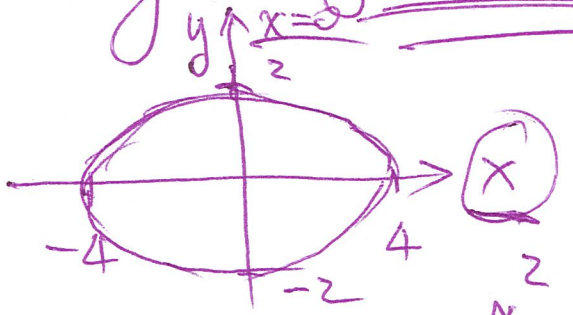
line z-axis

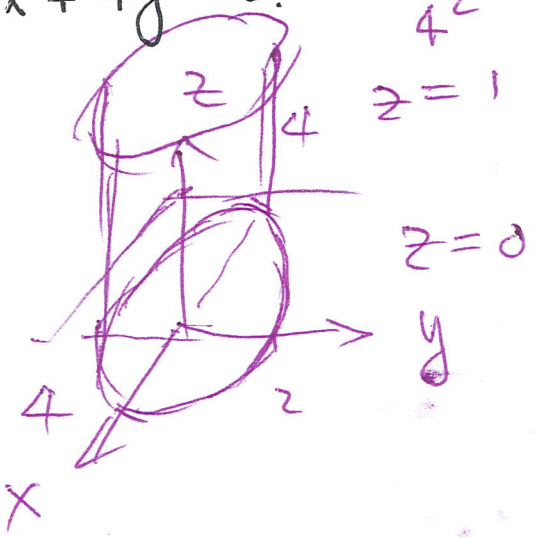
$x = 0$ and $y = 0$



Ex. 2 $x^2 + 4y^2 = 16$.

$\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1$

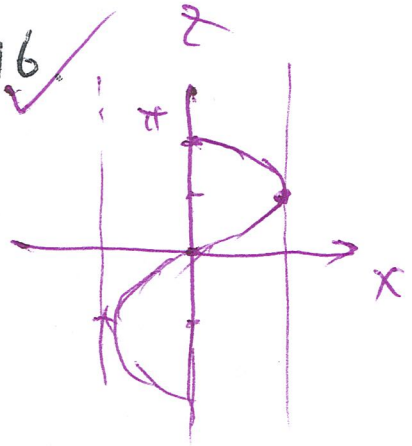




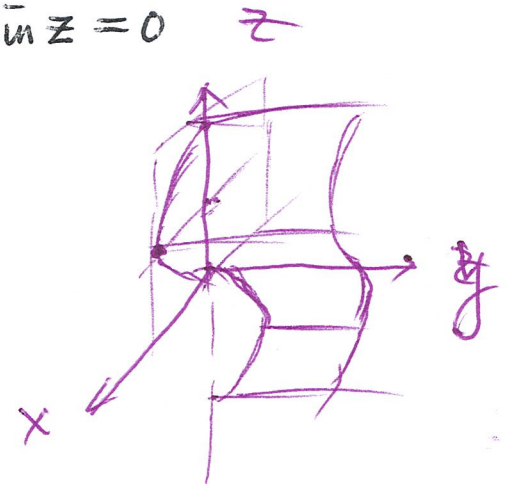
Lesson 3

Ex. 1 (Graphing cylinders) Sketch the graphs of the following cylinders in $\mathbb{R}^3$. Identify the axis to which each cylinder is parallel.

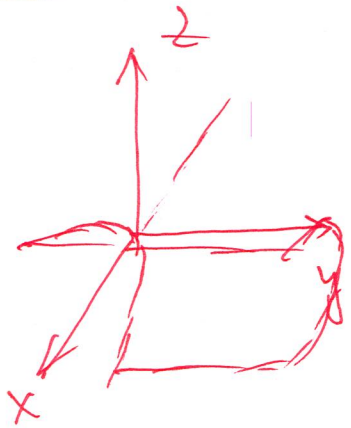
(a) $x^2 + 4y^2 = 16$



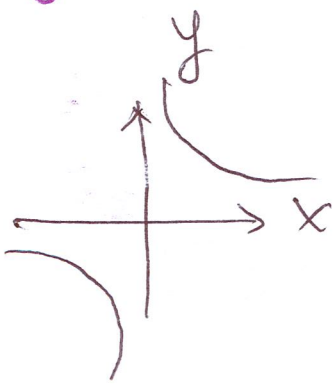
(b) $x - \sin z = 0$



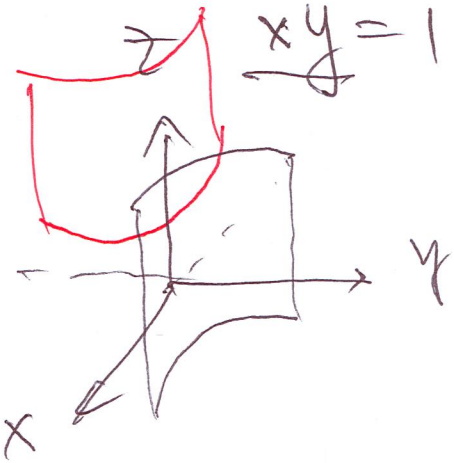
#12 (P863) $x - 2z^2 = 0$



#14 (P863) $x - \frac{1}{y} = 0$



$xy = 1$

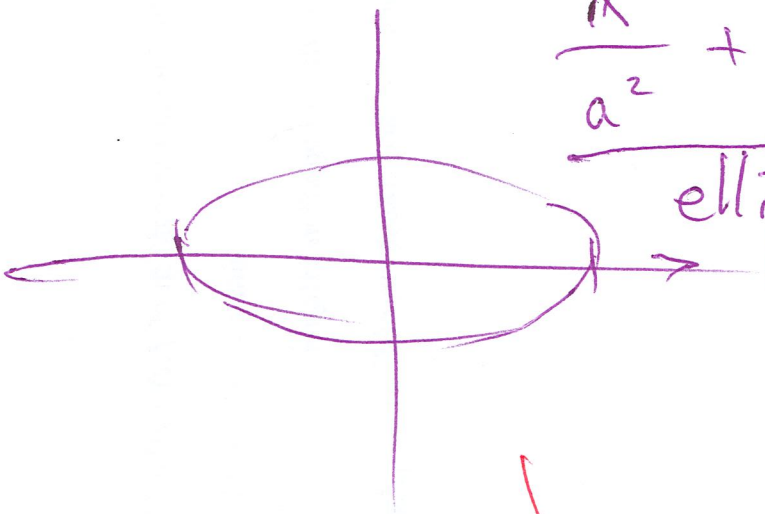


Quadric Surfaces

Quadric curves

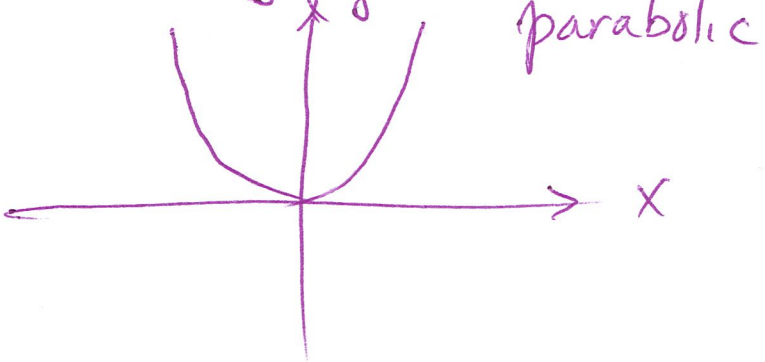
$$ax^2 + by^2 + cxy + dx + ey = f$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

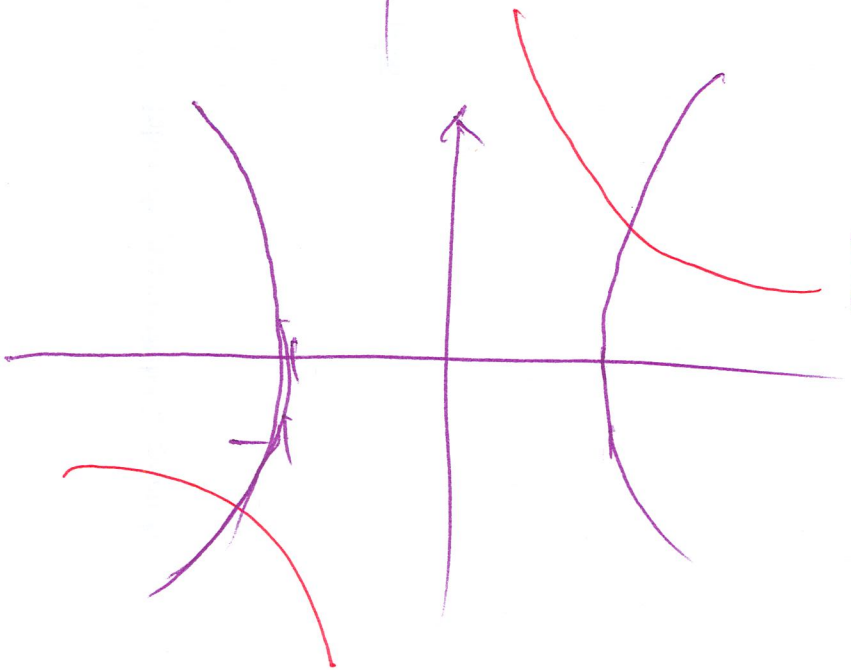


elliptic

$$y = ax^2$$



parabolic



$$x^2 - y^2 = 1$$

$y=0$: $x^2 = 1$

$x=0$ $\quad -y^2 = 1$

intercept with x-axis

y-

$$xy = 1$$

Quadratic Surfaces

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + \cdots = I$$

Ex. 2 (ellipsoid) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

(1) intercepts

(2) traces

x-axis $y=0$ and $z=0$ $\quad a=1,\ b=2,\ c=3$

$x^2 + 0 + 0 = 1 \Rightarrow x = \pm 1$

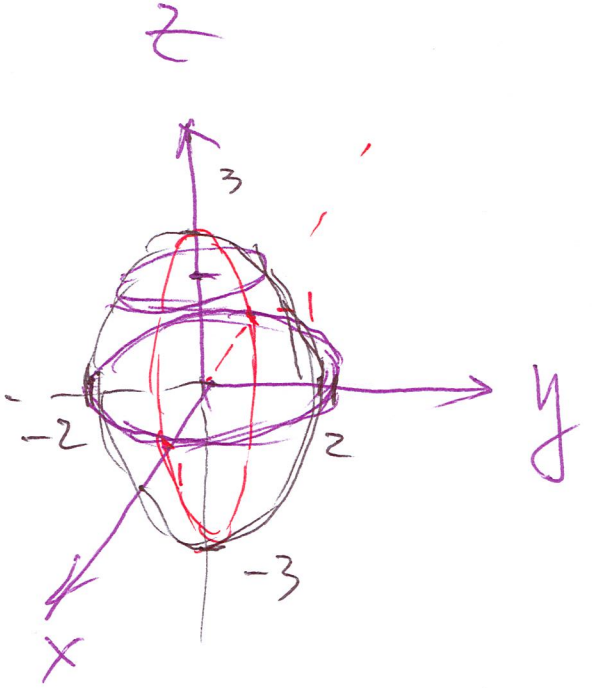
y-intercepts $x=0,\ z=0 \Rightarrow y^2 = 2^2 \Rightarrow y = \pm 2$

Traces

xy-plane : $z=0$ $\quad x^2 + \dfrac{y^2}{2^2} = 1$

xz-plane $y=0 \Rightarrow$ $\quad x^2 + \dfrac{z^2}{3^2} = 1$

yz-plane $x=0 \Rightarrow \dfrac{y^2}{2^2} + \dfrac{z^2}{3^2} = 1$



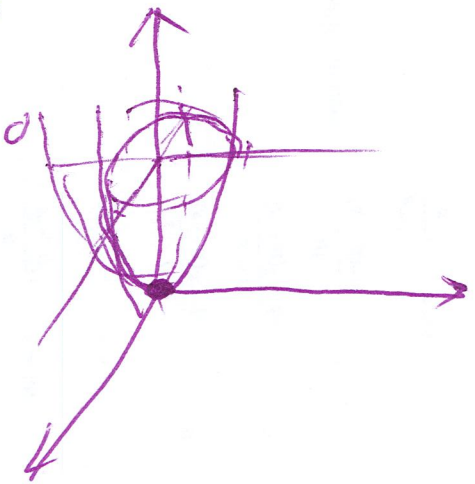
Ex. 3 (paraboloid) $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $a=1,\ b=1$

$= x^2 + y^2$

intercepts $x=0=y \Rightarrow z=0$

$x=0,\ z=0 \Rightarrow y=0$

~~$x=0$~~ $y=0,\ z=0 \Rightarrow x=0$



trace $z=0$ $x^2+y^2=0$

$z=1$ $x^2+y^2=1$

$x=0$ $z=y^2$

$y=0$ $z=x^2$