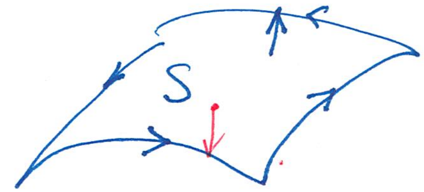
Stoke's Thrm

$$\iint_S \left(\nabla\times\vec{F}\cdot\vec{n}\right) dS = \oint_{\partial S} \vec{F}\cdot d\vec{r}$$

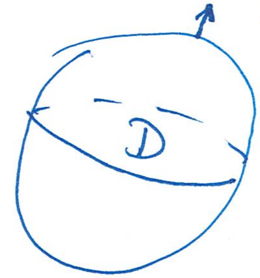
upward oriented S — ∂S oriented counter clockwise



$$\iint_{S_1} \nabla\times\vec{F}\cdot d\vec{S} = \iint_S \underline{\underline{\nabla\times\vec{F}}}\cdot d\vec{S} \qquad \oint_{\partial S} \vec{F}\cdot\vec{T}\, ds$$

$\partial S_1 = \partial S$

outward normal

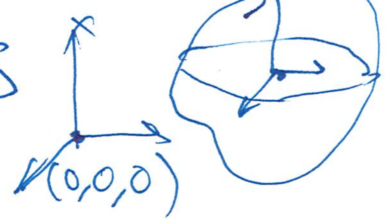


Divergence Thrm

$\vec{F} = \langle f, g, h \rangle$

$$\iiint_D \nabla\cdot\vec{F}\, dV = \iint_{\partial D} \vec{F}\cdot d\vec{S} = \iint_{\partial D} \left(\vec{F}\cdot\vec{n}\right) dS$$

$\nabla\cdot\vec{F} = \dfrac{\partial f}{\partial x} + \dfrac{\partial g}{\partial y} + \dfrac{\partial h}{\partial z}$



$\boxed{\cancel{\nabla\cdot\vec{F} = 0}}$

$$\vec{F} = \frac{\langle x, y, z\rangle}{|\vec{r}|^3} \qquad \iint_S \vec{F}\cdot\vec{n}\, dS = \begin{cases} \iiint_D \nabla\cdot\vec{F}\, dV = 0, & (0,0,0) \text{ not inside of } S \\ 4\pi & (0,0,0) \text{ inside of } S \end{cases}$$

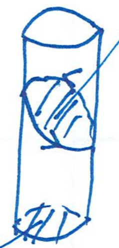
$|\vec{r}| = (x^2+y^2+z^2)^{\frac{1}{2}}$

S — any closed surface

#54

S

$x+y+z=1$



C

R $x^2+y^2=9$

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_S \cancel{\nabla\times} \nabla\times\vec{F}\cdot \vec{n}\, dS$$

$$= \iint_R \nabla\times\vec{F}\left(\vec{r}(x,y)\right)\cdot\langle 1,1,1\rangle\, dA$$

$$= \iint_R \langle 0,x^2,y^2\rangle\cdot\langle 1,1,1\rangle\, dA$$

$$= \iint_R (x^2+y^2)\, dA$$

$$= \int_0^3 r\,dr\int_0^{2\pi} r^2\, d\theta$$

$\vec{F}=\langle x^2 z,\ xy^2,\ z^2\rangle$

$S:\ z=1-x-y$, $(x,y)\in R:\ x^2+y^2\le 3^2$

$\vec{r}(x,y)=\langle x,\ y,\ 1-x-y\rangle$

$\vec{r}_x\times\vec{r}_y = \langle -\frac{\partial z}{\partial x},\ -\frac{\partial z}{\partial y},\ 1\rangle = \langle 1,1,1\rangle$ upward

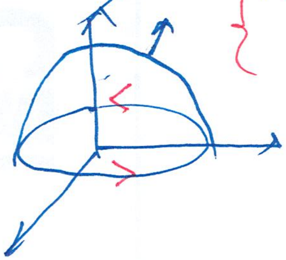
$$\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ x^2 z & xy^2 & z^2 \end{vmatrix} = \langle 0,\ x^2,\ y^2\rangle$$

#53

$$\iint_S \left(\nabla\times\vec{F}\cdot\vec{n}\right) dS = \oint_{\partial S}\vec{F}\cdot d\vec{r} = \int_0^{2\pi}\langle (2\cos t)^2,\ (2\sin t)^2,\ 0\rangle\cdot\langle -2\sin t,\ 2\cos t,\ 0\rangle\, dt$$

$$= 8\int_0^{2\pi}\left[-\cos^2 t\sin t+\sin^2 t\cos t\right]$$

$S:\ x^2+y^2+z^2=2^2$



$\begin{cases} z=0 \\ x^2+y^2=2^2 \end{cases}$

$\vec{F}=\langle x^2 e^{yz},\ y^2 e^{xz},\ z^2 e^{xy}\rangle$

$\vec{r}(t)=\langle 2\cos t,\ 2\sin t,\ 0\rangle$

$\vec{r}'(t)=\langle -2\sin t,\ 2\cos t,\ 0\rangle$

#52 $\iint_{\Sigma} \vec{F}\cdot\vec{n}\, dS = \iiint_{D} \nabla\cdot\vec{F}\, dV = 3\iiint_{D} dV$

$\vec{F} = \langle x, y, z\rangle$

$\nabla\cdot\vec{F} = 1+1+1 = 3$

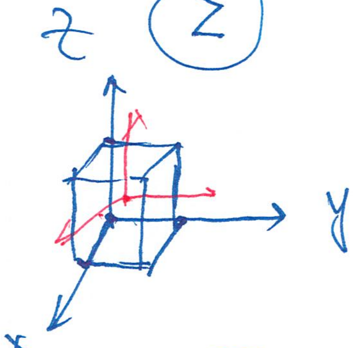
$= 3\,V(D)\ 3\cdot\frac{4}{3}\pi r^3\Big|_{r=1} = 4\pi$

#51 $\iint_{\Sigma} \vec{F}\cdot\vec{n}\, dS = \iiint_{D} \nabla\cdot\vec{F}\, dV = 0$

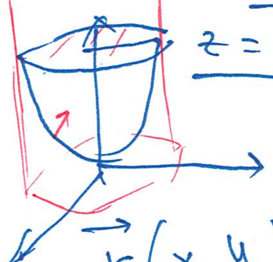
$\vec{F} = \langle \cos z, \sin z, xy\rangle$

$\nabla\cdot\vec{F} = 0+0+0 = 0$

$\cancel{4\pi}$



#50 $\Sigma : z = x^2+y^2 \quad z \le 4$



$z = 4$

$\vec{F} = \langle x, y, z\rangle$

$\vec{r}(x,y) = \langle x, y, x^2+y^2\rangle$

$\vec{r}_x \times \vec{r}_y = \langle -2x, -2y, 1\rangle$ upward

$\iint_{\Sigma} \vec{F}\cdot\vec{n}\, dS = \iint_{R} \vec{F}(\vec{r}(x,y))\cdot\vec{r}_x\times\vec{r}_y\, dA$

$= \iint_{R} \langle x, y, x^2+y^2\rangle\cdot\langle -2x, -2y, 1\rangle\, dA$

$= \iint_{R} \left[-2x^2 - 2y^2 + (x^2+y^2)\right] dA$

$= -\iint_{R} (x^2+y^2)\, dA = -\int_0^2 r\,dr \int_0^{2\pi} d\theta\, r^2$

$R: x^2+y^2 \le 4$

#49

$$\iint_\Sigma x\,dS$$

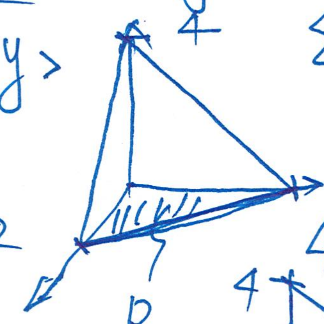
$$\Sigma: \ 2x+y+z=4 \quad \rightarrow \quad z = 4-2x-y$$

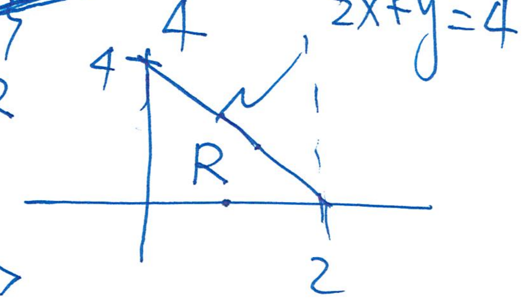
$$\vec r(x,y) = \langle x, y, 4-x-y \rangle$$

$$\vec r_x \times \vec r_y = \langle 2, 1, 1 \rangle$$

$$= \iint_R x \left| \vec r_x \times \vec r_y \right| dA$$

$$= \iint_R x\sqrt{6}\, dA = \int_0^2 dx \int_0^{4-2x} \sqrt{6}\, x\, dy$$





#48

$$A(R) = \iint_R 1\, dA \ \overset{g=x,\ f=0}{=} \ \oint_{\partial R} 0\,dx + x\,dy$$

$$\overset{g=0,\ f=-y}{=} \ \oint_{\partial R} -y\,dx$$

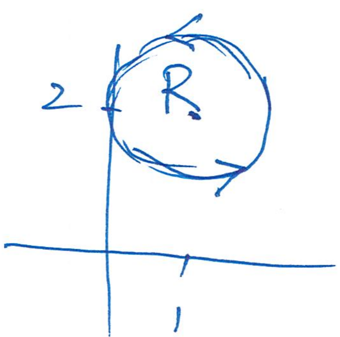
$$\vec F = \langle f, g \rangle$$

$$\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA = \oint_{\partial R} \vec F \cdot d\vec r$$

$$= \oint_{\partial R} f\,dx + g\,dy$$

#47

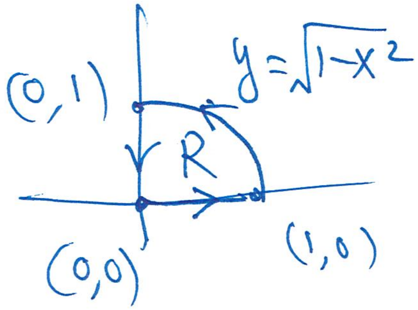
$$\oint_C \vec F \cdot d\vec r = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA = \iint_R \left[(2xy-1) - 2xy \right] dA$$



$$\vec F = \langle xy^2 - 1, x^2 y - x \rangle$$

$$= \iint_R (-1)\, dA = (-1)\, A(R) = -\pi$$

#46 $\int_C xy\,dy = \int_C \langle 0, xy \rangle \cdot d\vec{r}$



$$= \iint_R \left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}(0) \right] dA = \iint_R y\, dA$$

$$= \int_0^1 r\,dr \int_0^{\frac{\pi}{2}} d\theta \; r\sin\theta$$

$\vec{F} = \langle f, g, h \rangle$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C f\,dx + g\,dy + h\,dz$$