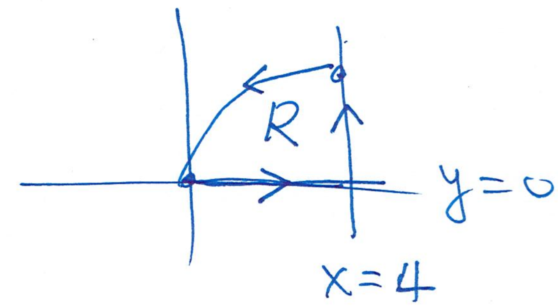
#45 $\int_C y^2 dx + 6xy\, dy$, C: boundary of $y=\sqrt{x}$, $y=0$, $x=4$; counterclockwise

$$= \int_C \langle y^2, 6xy \rangle \cdot d\vec{r} = \iint_R \nabla \times \vec{F}\, dA = 4 \iint_R y\, dA$$



$$\nabla \times \vec{F} = \frac{\partial}{\partial x}(6xy) - \frac{\partial}{\partial y}(y^2) \qquad = 4\int_0^4 \int_0^{\sqrt{x}} y\, dy\, dx$$

$$= 6y - 2y = 4y$$

#44 $\vec{F} = \langle x^3 + 2xy,\ x^2 - y^2 \rangle$ $\quad \nabla \times \vec{F} = \frac{\partial}{\partial x}(x^2 - y^2) - \frac{\partial}{\partial y}(x^3 + 2xy)$

$$= 2x - 2x = 0$$

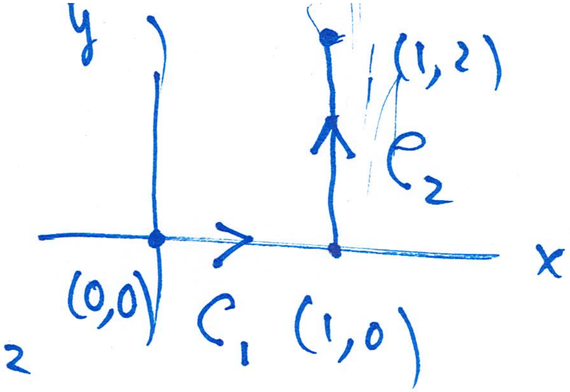
#43 $\int_C x\, dx + y\, dy + xy\, dz$, $C: \vec{r}(t) = \langle \cos t, \sin t, \cos t \rangle$, $-\frac{\pi}{2} \le t \le 0$.

$$\vec{r}\,'(t) = \langle -\overset{\sin t}{\cancel{\cos t}}, \cos t, -\sin t \rangle$$

$$= \int_C \langle x, y, xy \rangle \cdot d\vec{r} = \int_{-\frac{\pi}{2}}^{0} \langle \cos t, \sin t, \cos t \sin t \rangle \cdot \langle -\sin t, \cos t, -\sin t \rangle\, dt$$

$$= \int_{-\frac{\pi}{2}}^{0} \left(\cancel{-\cos t \sin t} + \cancel{\sin t \cos t} - \cos t \sin^2 t \right) dt$$

#42 $\vec{F} = \langle y, x^2 \rangle$, C: line segments $(0,0)\to(1,0)\to(1,2)$.



$$\int_C \vec{F}\cdot d\vec{r} = \int_C y\,dx + x^2\,dy = \underline{\int_{C_1} y\,dx + x^2\,dy} + \int_{C_2} y\,dx + x^2\,dy$$

$$= \int_{C_1} y\,dx + \int_{C_2} x^2\,dy = \int_{C_1} 0\,dx + \int_{C_2} 1^2\,dy = \int_0^2 1\,dy = 2$$

#41 C: $\underline{y = x^2}$, $0 \le x \le 2$, $\rho(x,y) = x$

$\vec{r}(x) = \langle x, x^2 \rangle$, $0 \le x \le 2$

$\vec{r}\,'(x) = \langle 1, 2x \rangle$, $|\vec{r}\,'(x)| = \sqrt{1+4x^2}$

$$m = \int_C \rho\,ds = \int_C x\,ds$$

$$= \int_0^2 x \underbrace{\sqrt{1+4x^2}\,dx}_{ds} \quad \frac{u = 1+4x^2 : 1 \to 17}{du = 8x\,dx} \quad \int_1^{17} \frac{1}{8} u^{\frac{1}{2}}\,du = \frac{1}{8}\cdot\frac{2}{3} u^{\frac{3}{2}} \Big|_1^{17}$$

#40

A. $\operatorname{curl}(\operatorname{grad} f) = \underline{\vec{0} = \nabla \times (\nabla f)}$ ✓

B. $\operatorname{div}(\operatorname{curl} \vec{F}) = \vec{0} = \nabla \cdot (\nabla \times \vec{F})$ ✓

C. $\operatorname{grad}(\operatorname{div} \vec{F}) = 0 = \nabla(\underline{\nabla \cdot \vec{F}})$ ✗

D. $\operatorname{curl} \vec{F} = \nabla \times \vec{F}$. ✓

E. $\operatorname{div} \vec{F} = \nabla \cdot \vec{F}$ ✓

#39 (1) $\vec{F} = \langle xy^2+x,\ x^2y-y^2 \rangle$, (2) $\vec{F} = \langle \frac{x}{y}, \frac{y}{x} \rangle$, (3) $\vec{F} = \langle ye^z,\ xe^z+e^y,\ (xy+1)e^z \rangle$

$\nabla\times\vec{F} = \frac{\partial}{\partial x}(x^2y-y^2) - \frac{\partial}{\partial y}(xy^2+x)$

$= 2xy - 2xy = 0$ ✓

$\nabla\times\vec{F} = \frac{\partial}{\partial x}(yx^{-1}) - \frac{\partial}{\partial y}(xy^{-1})$

$= y\cdot(-1)\cdot x^{-2} - x\cdot(-1)\,y^{-2}$

$= -\frac{y}{x^2} + \frac{x}{y^2} \neq 0$ ✗

$\nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ ye^z & xe^z+e^y & xye^z+e^z \end{vmatrix}$

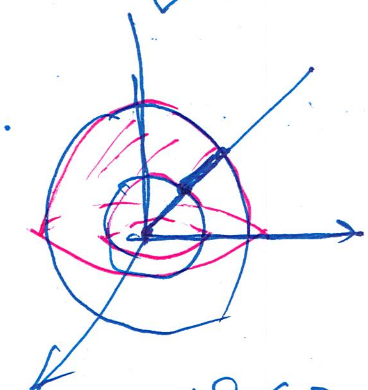
$= \langle xe^z - xe^z,\ ye^z - ye^z,\ e^z - e^z \rangle$

$= \langle 0, 0, 0 \rangle$ ✓

#38 $\iiint_D \sqrt{x^2+y^2+z^2}\,dV$, D: $z = \sqrt{4-x^2-y^2}$ and $z = \sqrt{1-(x^2+y^2)}$

$z^2 = 4-x^2-y^2$ $\qquad z^2 = 1-x^2-y^2$

$= \int_1^2 d\rho \int_0^{\frac{\pi}{2}} d\varphi \int_0^{2\pi} d\theta\ \rho \cdot \rho^2 \sin\varphi$



$1 \le \rho \le 2$

$0 \le \varphi \le \frac{\pi}{2}$

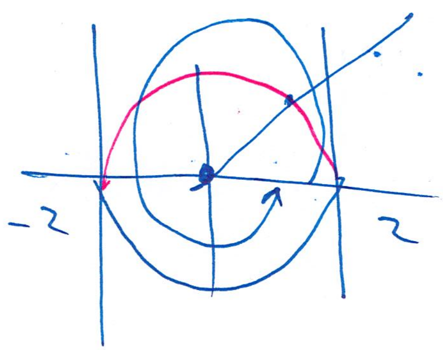
$0 \le \theta \le 2\pi$

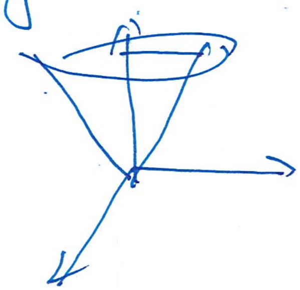
#37 $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} dz\,dy\,dx = \int_0^{2\pi}\int_0^{2}\int_{\sqrt{x^2+y^2}}^{2} dz\ \underline{\underline{r\,dr}}\,d\theta$

$\sqrt{x^2+y^2} \le z \le 2$

$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$

$-2 \le x \le 2$



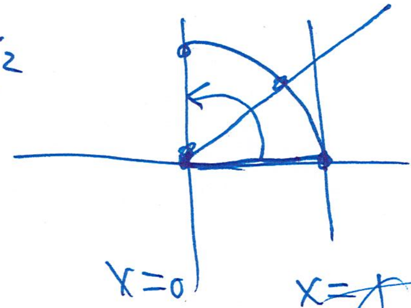


#36 $\int_0^1\int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^3\,dy\,dx = \int_0^1\int_0^{\frac{\pi}{2}} (r\sin\theta)^2(r^2)^3\,d\theta\,r\,dr$

$r^2\cdot r^3 = r^5$

$0\le y\le\sqrt{1-x^2}$

$0\le x\le 1$

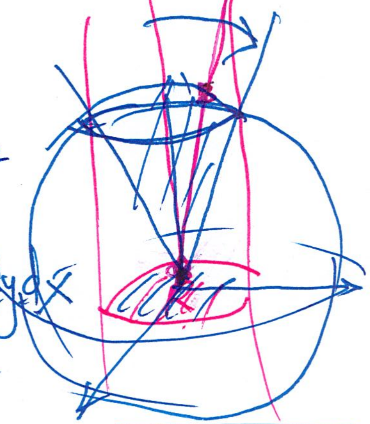


$= \int_0^1\int_0^{\frac{\pi}{2}} r^9\sin\theta\,d\theta\,dr$

$(r^2)^3 = r^6$

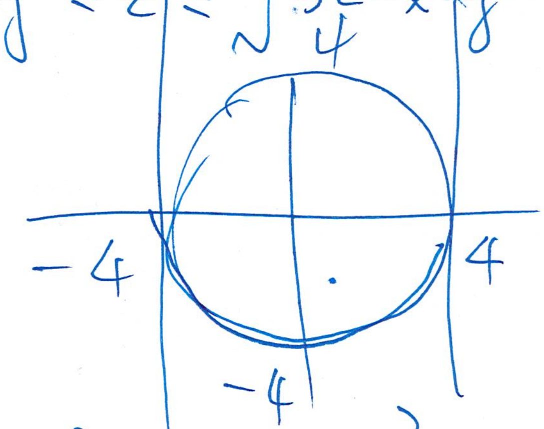
#34 above by $x^2+y^2+z^2=32$, below by $z^2=x^2+y^2$. $\rho(x,y,z)=z$

$$m=\iiint_D \rho\,dV = \iint_R \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dA = \int_{-4}^{4}\int_{-\sqrt{4^2-x^2}}^{\sqrt{4^2-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$$



$\sqrt{x^2+y^2}\le z\le\sqrt{32-x^2-y^2}$

$32=x^2+y^2+z^2=2(x^2+y^2)\Rightarrow x^2+y^2=16=4^2$



#35 $m=\iiint_D z\,dV = \int_0^{4\sqrt{2}} d\rho\int_0^{\frac{\pi}{4}} d\varphi\int_0^{2\pi} d\theta\;\underbrace{\rho\cos\varphi}_{z}\;\rho^2\sin\varphi$

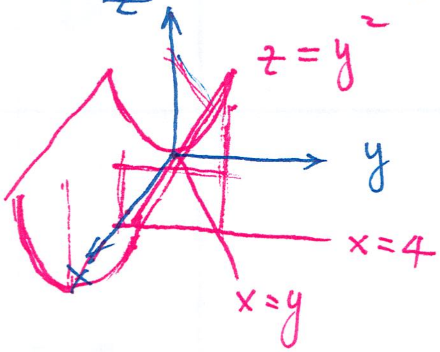
$\rho^2=32$

$\rho=\sqrt{32}=4\sqrt{2}$

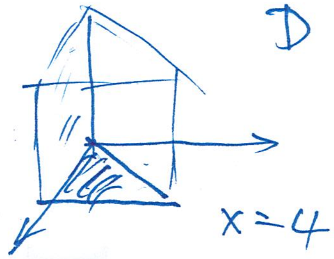
$(\rho\cos\varphi)^2=z^2=x^2+y^2=r^2=(\rho\sin\varphi)^2$

$\cancel{\rho}\cos\varphi=\cancel{\rho}\sin\varphi \qquad \frac{\sin\varphi}{\cos\varphi}=1\Rightarrow\varphi=\frac{\pi}{4}$

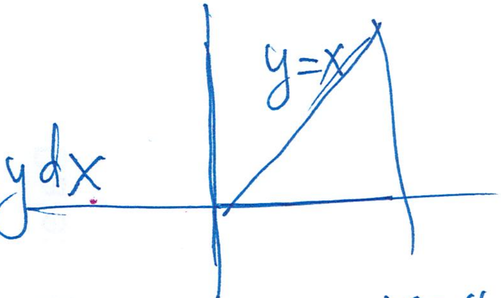
#33 $z=y^2,\ y=x,\ y=0,\ z=0,\ x=4$ in 1^{st} octant



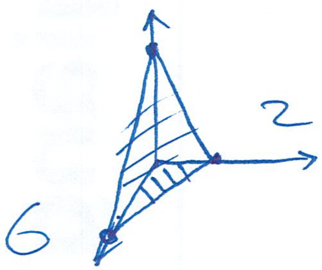
$$V=\iiint_D dV=\iint_R\int_0^{y^2} dz\,dA$$



$$=\int_0^4\int_0^x\int_0^{y^2} dz\,dy\,dx$$

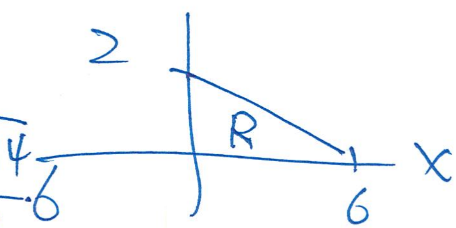


#32 $x+3y+2z=6$



$$A=\iint_S dS=\iint_R\sqrt{\left(\tfrac{1}{2}\right)^2+\left(\tfrac{3}{2}\right)^2+1}\,dA$$

$$z=\frac{1}{2}(6-x-3y)\qquad =\sqrt{\frac{1}{4}+\frac{9}{4}+1}\ A(R)=\frac{\sqrt{14}}{2}6$$



$$\vec{r}(x,y)=\langle x,\ y,\ \tfrac{1}{2}(6-x-3y)\rangle,\ \vec{r}_x\times\vec{r}_y=\langle\tfrac{1}{2},\tfrac{3}{2},1\rangle$$

#31 $r=5\sin 3\theta$

$\sin 3\theta=0\Rightarrow\theta=0,\frac{\pi}{3},$ $A=\displaystyle\int_0^{\frac{\pi}{3}}\int_0^{5\sin 3\theta} r\,dr\,d\theta$