Ex.4 (hyperboloid of one sheet)

$$\frac{x^2}{4}+\frac{y^2}{9}-z^2=1$$

intercepts

x-axis $(y=0=z)$ $\frac{x^2}{4}=1 \Rightarrow x=\pm 2$

y-axis $(x=0=z)$ $y^2/9=1 \Rightarrow y=\pm 3$

z-axis $(x=0,\ y=0)$ $-z^2=1 \Rightarrow$ no intercept

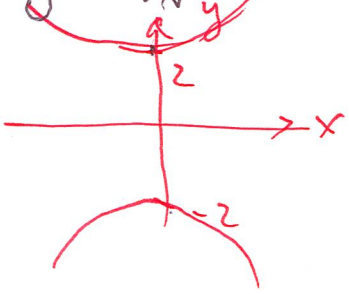
traces xy-plane $(z=0)$ $\frac{x^2}{4}+\frac{y^2}{9}=1$

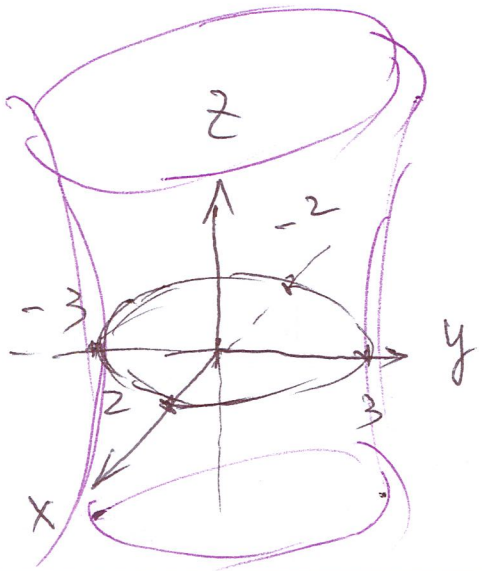
$z=t$ $\frac{x^2}{4}+\frac{y^2}{9}=1+t^2$

$$\frac{x^2}{4(1+t^2)}+\frac{y^2}{9(1+t^2)}=1 \Rightarrow$$

$x=\pm 2\sqrt{1+t^2}$

$y=\pm 3\sqrt{1+t^2}$





Ex.5 (hyperbolic paraboloid)

$$z=x^2-\frac{y^2}{4}$$

intercepts $x=0=y \Rightarrow z=0$; $y=z=0 \Rightarrow x=0$

$x=0=z \Rightarrow y=0$ $(0,0,0)$

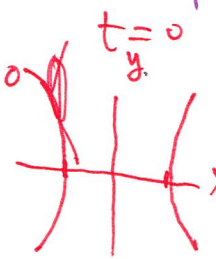
traces $(y=0,$ xz-plane$)$ $z=x^2$

$(x=0,$ yz-plane$)$ $z=-\frac{y^2}{4}$

$z=t$ $x^2-\frac{y^2}{4}=t>0$

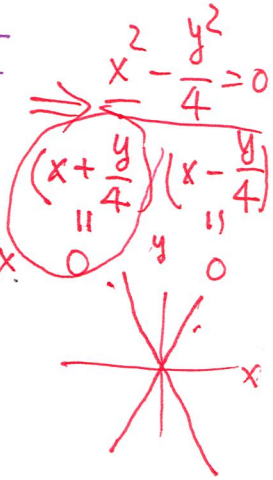
$t=1$ $x^2-\frac{y^2}{4}=1$

$t=-1$ $x^2-\frac{y^2}{4}=-1$

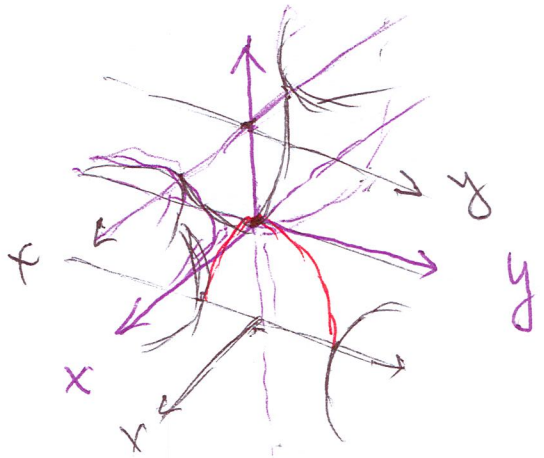


$x^2-\frac{y^2}{4}=0$

$\left(x+\frac{y}{4}\right)\left(x-\frac{y}{4}\right)=0$



z



Ex. 6 (elliptic cone)

$$\frac{y^2}{4} + z^2 = 4x^2$$

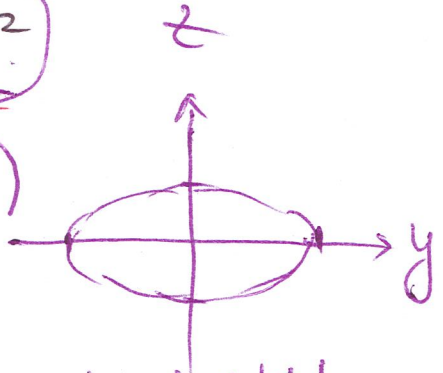
intercepts $(0, 0, 0)$

traces $x = t$

$$\frac{y^2}{4t^2} + \frac{z^2}{4t^2} = 1$$

$$y = \pm 4|t|$$

$$z = \pm 2|t|$$

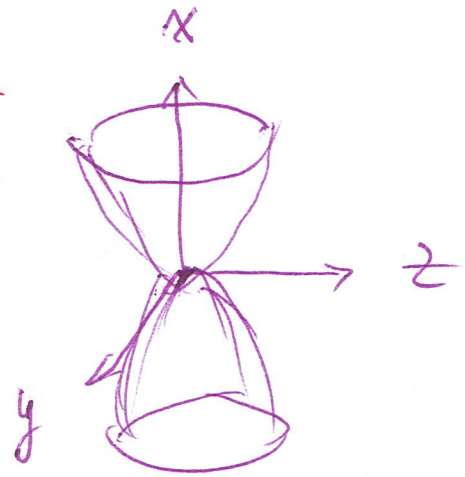


~~$x = 0$~~

~~$x = t$~~

~~$z = k$~~

$$4x^2 - \frac{y^2}{4} = k^2$$



Ex. 7 (hyperboloid of two sheets)

$$-16x^2 - 4y^2 + z^2 + 64x - 80 = 0$$

$$-16(x^2 - 4x + 4 - 4) - 4y^2 + z^2 = 80$$

$$-16(x-2)^2 - 4y^2 + z^2 = 80 - 64 = 16$$

$$-(x-2)^2 - \frac{y^2}{2^2} + \frac{z^2}{4^2} = 1$$

$\hat{x} = x - 2$

$$-\hat{x}^2 - \frac{y^2}{2^2} + \frac{z^2}{4^2} = 1$$

$z = \pm 4$

intercepts **no** intercept with x-axis, y-axis

traces $z = t$ $\quad |t| > 4$

$$\hat{x}^2 + \frac{y^2}{2^2} = \frac{t^2}{4^2} - 1 = \frac{t^2 - 4^2}{4^2}$$

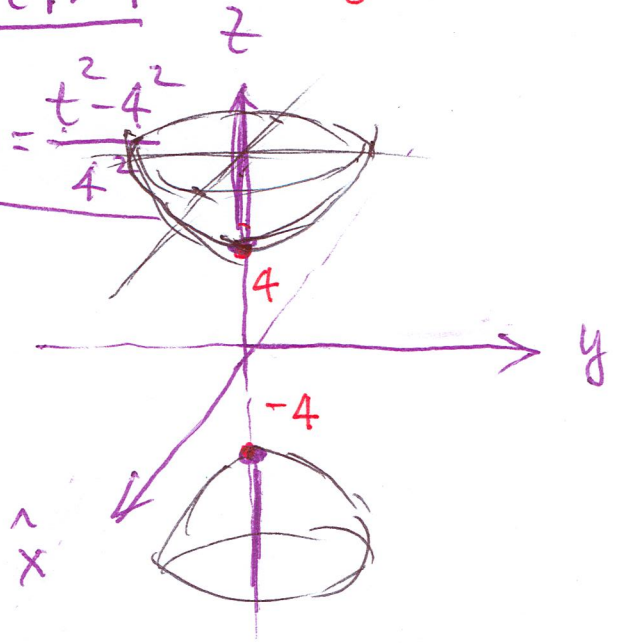
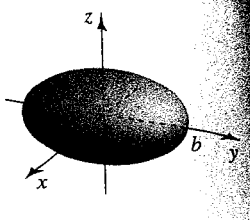
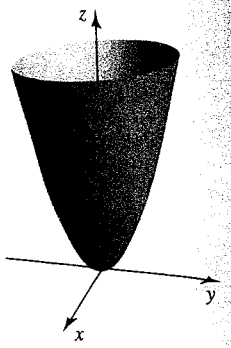
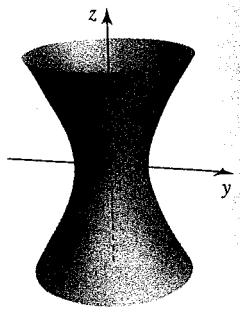
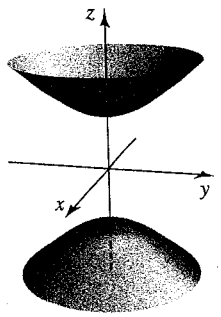
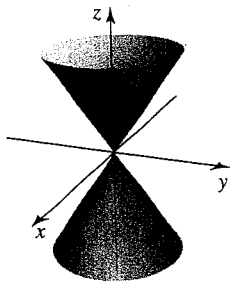


Table 13.1

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	All traces are ellipses.	
Elliptic paraboloid	$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	
Hyperboloid of two sheets	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Traces with $z = z_0$ with $\lvert z_0\rvert > \lvert c\rvert$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$	Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.