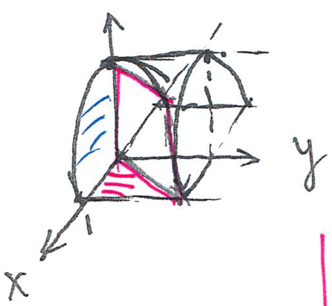
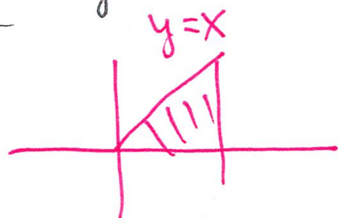
#30 above by $z=1-x^2$, below by xy-plane, sides by $y=0$, $y=x$ in 1^{st} octant



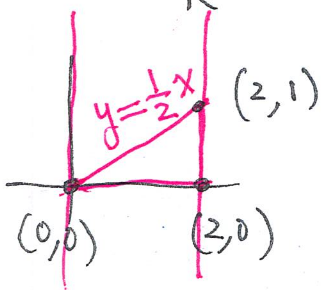
$$V=\iiint_D dV=\iint_R \int_0^{1-x^2} 1\,dz\,dA=\iint_R (1-x^2)\,dA$$



$$=\int_0^1\int_0^x (1-x^2)\,dy\,dx=\int_0^1 x(1-x^2)\,dx$$

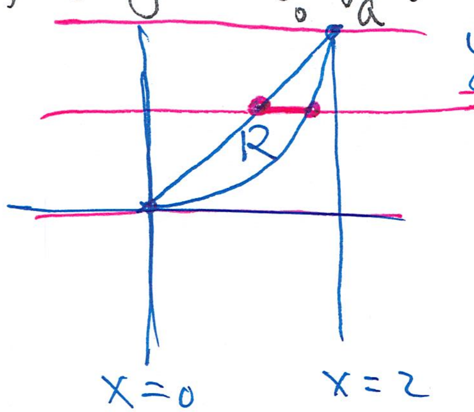
#29 $\iint_R y\,dA$, $=\int_0^2 dx\int_0^{\frac{1}{2}x} y\,dy$

$x=1$



#28 $\int_0^2\int_{x^2}^{2x} f(x,y)\,dy\,dx=\int_0^4\int_a^b f(x,y)\,dx\,dy$, $a=?$, $b=?$

$$\begin{cases} x^2\le y\le 2x \\ 0\le x\le 2\end{cases}$$



$y=2x$

$\Downarrow$

$x=\frac{1}{2}y$

$y=x^2 \Longrightarrow x=\sqrt{y}$

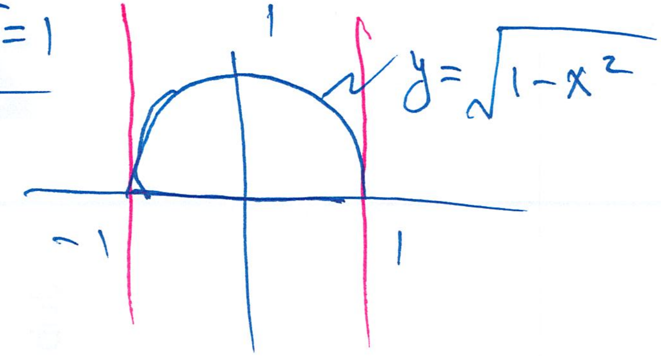
$0\le y\le 4$

$$\int_0^4 dy\int_{\cancel{2x}\ \frac{1}{2}y=a}^{\sqrt{y}=b} f\,dx$$

$\frac{1}{2}y\le x\le\sqrt{y}$

#27 $\iint_R f(x,y)\,dA$, $R: x^2+y^2 \le 1$, $y \ge 0$

$x^2+y^2=1$



$$= \int_{-1}^{1} dx \int_0^{\sqrt{1-x^2}} f(x,y)\,dy$$

#26 $\int_1^3 \int_0^x \frac{1}{x}\,dy\,dx = \int_1^3 \frac{1}{x} \cdot x\,dx = \int_1^3 dx = 3-1=2$

#25 $\min_{xy=1} f(x,y) = 4x^2+y^2$

$xy=1 \Rightarrow x = \frac{1}{y}$

$f(x,y) = 4\left(\frac{1}{y}\right)^2 + y^2 = g(y)$

$\min_y g(y)$, $0 = g'(y) = \left(4y^{-2}+y^2\right)' = -8y^{-3}+2y$

$= 2y^{-3}\underbrace{\left[-4+y^4\right]}_{=0}$

$a^2-b^2=(a+b)\cdot(a-b)$

$0 = y^4-4 = (y^2-2)(y^2+2)$

$= (y-\sqrt{2})(y+\sqrt{2})(y^2+2)$

$y = -\sqrt{2}, \sqrt{2}$

$g(-\sqrt{2}) = g(\sqrt{2}) = 4 \cdot \frac{1}{2} + 2 = 4$

$f = 4x^2+y^2$, $\nabla f = \langle 8x, 2y\rangle$

$g = xy$, $\nabla g = \langle y, x\rangle$

$\begin{cases} \nabla f = \lambda \nabla g \\ xy=1 \end{cases} \Rightarrow \dfrac{8x = \cancel{\lambda} y}{2y = \cancel{\lambda} x} \Rightarrow 8x^2 = 2y^2$

$y^2 = 4x^2$

$y = \pm 2|x| \to \pm 2x$

$xy=1$ $\quad 1 = xy = \pm 2x^2$

$\Rightarrow 2x^2 = 1 \Rightarrow x = \pm\sqrt{\frac{1}{2}}$

#24 $f(x,y) = 2x^3 - 6xy - 3y^2$

(1) critical pts:

$\langle 0,0\rangle = \nabla f = \langle \underbrace{6x^2 - 6y}_{= f_x}, \underbrace{-6x - 6y}_{= f_y} \rangle$

$= 6\langle x^2 - y, -x - y\rangle$

$\begin{cases} x^2 - y = 0 \\ x + y = 0 \Longrightarrow x = -y \end{cases} \Longrightarrow (-y)^2 - y = 0$

$0 = y^2 - y = y(y-1)$

$\Longrightarrow y = 0, 1$ $\quad (0,0), (-1,1)$

$x = -y = 0, -1$

(2) classifying critical pts.

$D(x,y) = f_{xx} f_{yy} - f_{xy}^2$

$= 12x \cdot (-6) - (-6)^2 = -6^2 \cdot 2x - 6^2$

$= -6^2(2x+1)$

$D(0,0) = -6^2 < 0 \Longrightarrow$ saddle

$D(-1,1) = -6^2(-2+1) = 6^2 > 0 \Longrightarrow$ loc. max.

$f_{xx}(-1,1) = 12x\Big|_{(-1,1)} = -12 < 0$

$f(x,y) = x^2 + y^2$

$D = 2 \cdot 2 - 0^2 = \underline{4 > 0}$

$f_{xx} = \underline{2 > 0}$

#23 $f(x,y,z) = \cancel{\;} x e^{y^2 - z^2}$

$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$

$= e^{y^2 - z^2} dx + x e^{y^2 - z^2} \cdot 2y\, dy + x e^{y^2 - z^2} (-2z) dz$

#22 $z = f(x,y) = \pi + \sin(\pi x^2 + 2y)$ at $(x,y) = (2, \pi)$

$z = f(2,\pi) = \pi + \sin(4\pi + 2\pi) = \pi$

$(\pi x^2 + 2y)\Big|_{(2,\pi)} = 4\pi + 2\pi = 6\pi$

$g(x,y,z) = z - \pi - \sin(\pi x^2 + 2y) = 0$

$\nabla g = \langle -\pi 2x \cos(\pi x^2 + 2y), -2\cos(\pi x^2 + 2y), 1\rangle$

$\nabla g(2,\pi,\pi) = \langle -4\pi, -2, 1\rangle$

$0 = \langle -4\pi, -2, 1\rangle \cdot \langle x-2, y-\pi, z-\pi\rangle$

$z = f(2,\pi) + \frac{\partial f}{\partial x}(2,\pi)(x-2) + \frac{\partial f}{\partial y}(2,\pi)(y-\pi)$

$y = f(a) + f'(a)(x-a)$

#21 $\underbrace{x^2+2y^2+3z^2}_{\overset{\shortparallel}{g(x,y,z)}}=6$ at $(1,1,-1)$

$\nabla g = \langle 2x, 4y, 6z \rangle = 2\langle x, 2y, 3z \rangle$

$\nabla g(1,1,-1) = 2\langle 1, 2, -3 \rangle$

$0 = \langle 1, 2, -3 \rangle \cdot \langle x-1, y-1, z+1 \rangle = (x-1) + 2(y-1) - 3(z+1)$

$= x + 2y - 3z + (-1 - 2 - 3)$

#20 $\underbrace{2\cos(\pi x y)}_{\overset{\shortparallel}{g(x,y)}} = \frac{1}{2}$ at $(\frac{1}{6}, 2)$

$\nabla g(\frac{1}{6}, 2) = \langle -\pi y \sin(\pi x y), -\pi x \sin(\pi x y) \rangle \Big|_{(\frac{1}{6},2)} = -\pi \langle 2\sin\frac{\pi}{3}, \frac{1}{6}\sin\frac{\pi}{3} \rangle$

$= -\pi \sin\frac{\pi}{3} \langle 2, \frac{1}{6} \rangle$

$= -6\pi \sin\frac{\pi}{3} \langle 12, 1 \rangle$

#19 $f(x,y,z) = 3xy - 9xz^2 + y$

$\nabla f = \langle 3y - 9z^2, 3x+1, -18xz \rangle$

$\nabla f(1,1,0) = \langle 3, 3+1, \cancel{-18}\,0 \rangle$

$\langle 3, 4, \cancel{-18}\,0 \rangle$

$|\nabla f(1,1,0)| = \sqrt{9+16} = 5$

#18 $f(x,y)=x^2y$ find $\vec{u}=\langle u_1,u_2\rangle$ s.t. $0=D_{\vec{u}}f(2,3)=\nabla f(2,3)\cdot\vec{u}$

$$=\langle 2xy, x^2\rangle\Big|_{(2,3)}\cdot\langle u_1,u_2\rangle$$

$$=\langle 12,4\rangle\cdot\langle u_1,u_2\rangle$$

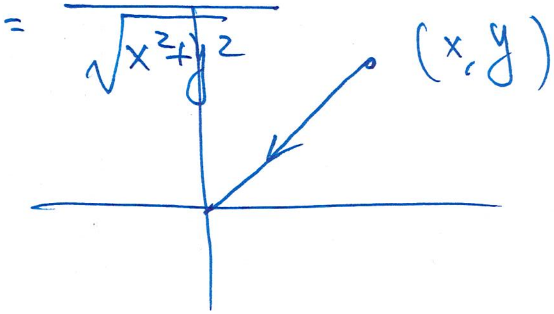
$$=4\langle 3,1\rangle\cdot\langle u_1,u_2\rangle=4(3u_1+u_2)$$

$\Rightarrow \boxed{3u_1+u_2=0}$ $u_2=-3u_1$

$\vec{u}=\langle 1,-3\rangle$

$$\frac{\langle 1,-3\rangle}{\sqrt{1^2+3^2}}=\frac{\langle 1,-3\rangle}{\sqrt{10}}$$

$$\vec{u}=\frac{-\langle x,y\rangle}{\sqrt{x^2+y^2}}$$



#17 $f(x,y)=5-4x^2-3y$ at

$$\nabla f\cdot\vec{u}=\langle -8x,-3\rangle\cdot\frac{\langle x,-y\rangle}{\sqrt{x^2+y^2}}=\frac{8x^2+3y^2}{\sqrt{x^2+y^2}}$$

#16 $f(x,y)=xy^2$, $\nabla f(2,3)=$