vector-valued function $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$$\lim_{t\to t_0} \vec{r}(t) = \langle \lim_{t\to t_0} x(t), \lim_{t\to t_0} y(t), \lim_{t\to t_0} z(t) \rangle$$

$$\vec{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

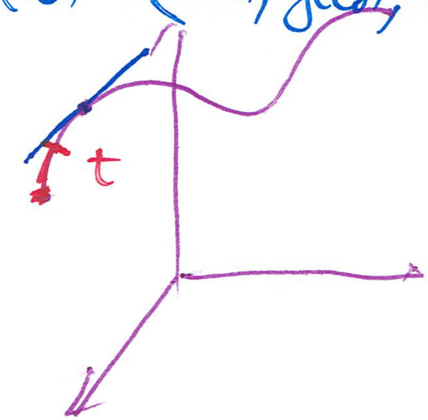
$$\left[f(t)\,\vec{r}(t)\right]', \quad \left[\vec{r}_1(t)\cdot\vec{r}_2(t)\right]', \quad \left[\vec{r}_1 \times \vec{r}_2\right]'$$

$$\frac{d}{dt} f\left(\vec{r}(t)\right) = ? \qquad \frac{d}{dt}\vec{r}\left(f(t)\right) = \vec{r}'(f(t))\,f'(t)$$

$$\int \vec{r}(t)\,dt = \langle \int x(t)\,dt, \int y(t)\,dt, \int z(t)\,dt \rangle$$

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle \quad t \in [a,b]$

$P_0 = \vec{r}(t_0) = (x(t_0), y(t_0), z(t_0))$



$\{ (x(t), y(t), z(t)) \mid t \in [a,b] \}$

parametric representation of C

$(x(t_0), y(t_0), z(t_0)) \in L$

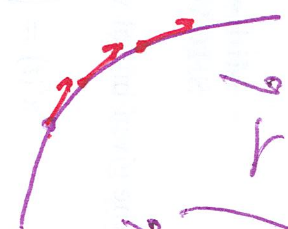
tangent equation

$\vec{r}\,'(t_0) \parallel L$

$$\Longrightarrow \boxed{\vec{l}(t) = \vec{r}(t_0) + (t - t_0)\,\vec{r}\,'(t_0)}$$

arc length $\quad L = \int_a^b |\vec{r}\,'(t)|\,dt \qquad s(t) = \int_a^t |\vec{r}\,'(u)|\,du = f(t)$

$t = f^{-1}(s)$



$\vec{r}(f^{-1}(s))$

curvature $\quad \kappa = \left| \frac{d\vec{T}}{ds} \right| = \frac{\left| \frac{d\vec{T}}{dt} \right|}{|\vec{r}\,'(t)|} = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$

$\vec{T} = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

$$\frac{d\vec{T}}{ds} = \frac{d\vec{T}}{dt} \cdot \frac{dt}{ds} = \frac{d\vec{T}}{dt} \Big/ \frac{ds}{dt}$$

$\vec{r}(t)$ — position

$\vec{v}(t) = \vec{r}\,'(t)$

$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$

- circular/sphere motion

$|\vec{r}(t)| = \text{const} \Longrightarrow \vec{r}(t) \perp \vec{v}(t)$

- $\vec{a} = \langle 0, -g \rangle$, $\vec{a} = \langle 0, 0, -g \rangle$

Lesson 9

Chapter 15 Functions of Several Variables (9 lectures)

$\underline{y = f(x)}$

$\vec{r}(t)$

§15.1 Functions of Several Variables

- functions of two variables

$$\boxed{z = f(x, y)}$$

single valued

domain $D = \{(x, y) \in \mathbb{R}^2 \mid f(x, y) \text{ is well defined}\}$

range $R = \{f(x, y) \mid (x, y) \in D\}$

examples find the domain of the following functions

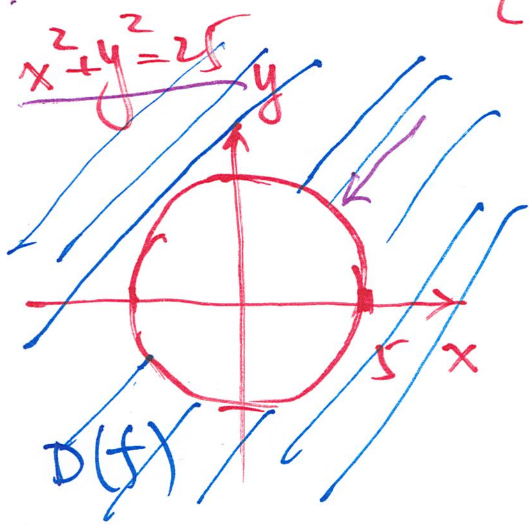
#18 $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-25}}$ ≥ 0 ; $-\infty$ $>$

$\begin{cases} \sqrt{x^2+y^2-25} \neq 0 \Rightarrow x^2+y^2-25 \neq 0 \\ x^2+y^2-25 \geq 0 \end{cases}$

$\Rightarrow \underline{x^2+y^2-25 > 0}$

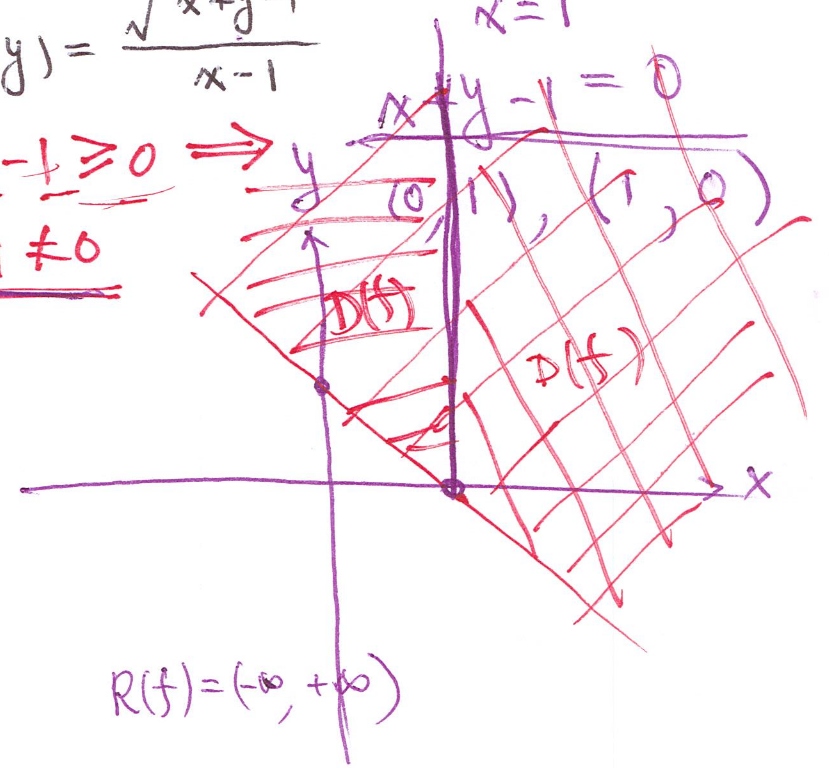
$\underline{x^2+y^2 > 25}$

$R(f) = [0, +\infty)$



$f(x,y) = \dfrac{\sqrt{x+y-1}}{x-1}$ $< +\infty$

$\begin{cases} x+y-1 \geq 0 \Rightarrow y \\ \underline{x-1 \neq 0} \end{cases}$

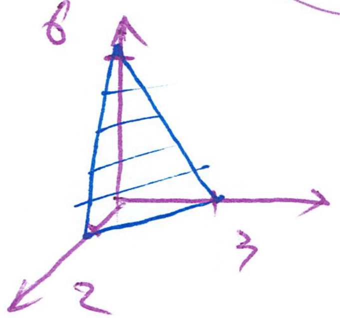


$R(f) = (-\infty, +\infty)$

• graph of $z=f(x,y)$ $\quad \{(x,y,f(x,y)) \mid (x,y)\in D(f)\}$

examples sketch the graph

$z=f(x,y)=6-3x-2y$;



$x=y=0 \Rightarrow z=6$

$x=z=0 \Rightarrow 6-2y=0 \Rightarrow y=3$

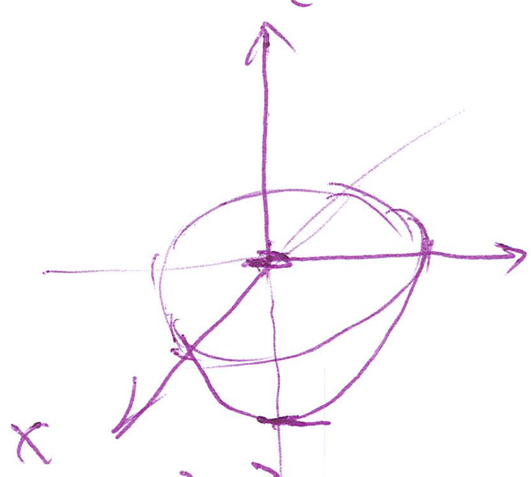
$y=z=0 \Rightarrow x=2$

$D(f)=R^2$

$R(f)=R=(-\infty,+\infty)$

#29 $z=-\sqrt{9-x^2-y^2}$

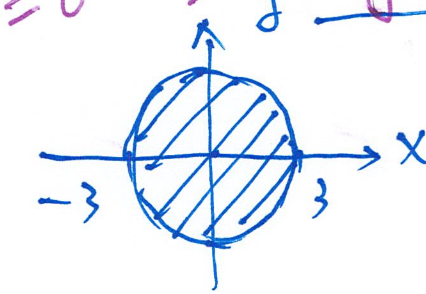
$\Rightarrow z^2=9-x^2-y^2 \Rightarrow x^2+y^2+z^2=9$



$R(f)=[-3,0]$

$9-x^2-y^2\geq 0 \Rightarrow x^2+y^2\leq 3^2$

$x^2+y^2=3^2$



• <u>level curve</u> $\begin{cases} z = f(x,y) \\ z = z_0 \end{cases} \Longrightarrow f(x,y) = z_0 \qquad \{(x,y)\in \mathbb{R}^2 \mid f(x,y) = z_0\}$

<u>examples</u> sketch level curve of

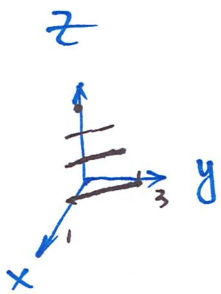
$z = \underline{f(x,y)} = 6-3x-2y$;

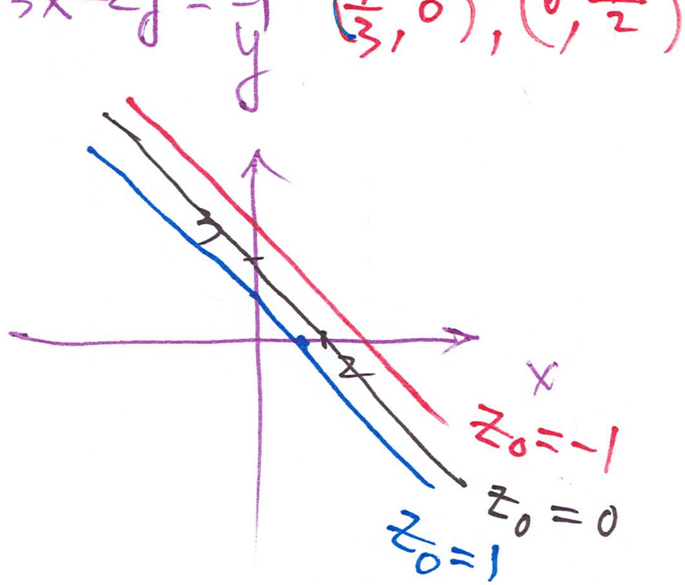
$\{(x,y) \mid 6-3x-2y = z_0\}$

$\underline{z_0 = 0}$ $\quad 6-3x-2y = 0 \quad (0,3)$ and $(2,0)$

$\underline{z_0 = 1}$ $\quad 6-3x-2y = 1 \quad \left(\frac{5}{3},0\right), \left(0,\frac{5}{2}\right)$

$\underline{z_0 = -1}$ $\quad 6-3x-2y = -1 \quad \left(\frac{7}{3},0\right), \left(0,\frac{7}{2}\right)$





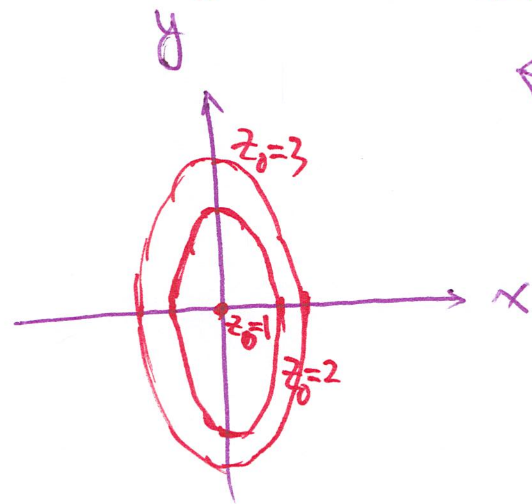
$z = h(x,y) = 4x^2 + y^2 + 1$

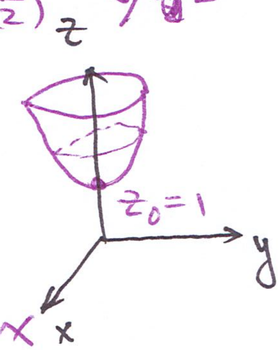
$\{(x,y) \mid 4x^2+y^2+1 = z_0\}$

$z_0 = 1 \Rightarrow 4x^2+y^2 = 0 \Longrightarrow (x,y) = (0,0)$

$z_0 = 2 \Rightarrow 4x^2+y^2 = 1 \Longrightarrow \frac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = 1$

$z_0 = 3 \Rightarrow 4x^2+y^2 = 2 \Longrightarrow \frac{x^2}{\left(\frac{1}{2}\right)} + y^2/2 = 1$





- functions of three variables

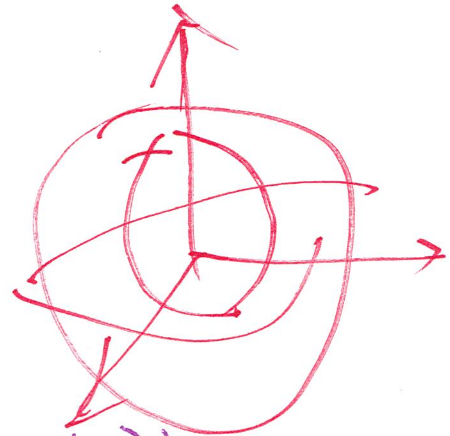
$w = f(x, y, z)$

domain

range

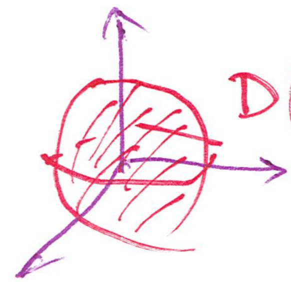
graph

level surface $\begin{cases} w = f(x,y,z) \\ w = w_0 \end{cases}$



example $w = f(x,y,z) = \sqrt{1-x^2-y^2-z^2} \geq 0$

$1-x^2-y^2-z^2 \geq 0 \Longrightarrow x^2+y^2+z^2 \leq 1$



$0 \leq w \leq 1$

$\sqrt{1-(x^2+y^2+z^2)} = w_0$

$w_0 = 0 \quad x^2+y^2+z^2 = 1$

$w_0 = 1 \quad x^2+y^2+z^2 = 0 \Rightarrow (0,0,0)$

$w_0 = \frac{1}{2}$

$1-(x^2+y^2+z^2) = \frac{1}{4}$

$\Rightarrow x^2+y^2+z^2 = 1 - \frac{1}{4} = \frac{3}{4}$