MA 26100
EXAM 1 Green
September 30, 2019

NAME ____________________ YOUR TA'S NAME ____________________

STUDENT ID # ____________________ RECITATION TIME ____________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): **00**

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **11** questions, each worth 9 points (you will automatically earn 1 point for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–11. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____________________

STUDENT SIGNATURE: ____________________

1. Find the equation of the plane through the point $(0, 1, 2)$ perpendicular to the planes given by $x - y + 2z = 1$ and $3x + 2z = -4$

A. $y + 2z = 10$

B. $2x - 4y - 3z = -10$

C. $y - 2z = 2$

D. $-2x - 4y + 3z = 2$

E. $4x - y + 4z = -3$

$\vec{n}_1 = \langle 1, -1, 2 \rangle$, $\vec{n}_2 = \langle 3, 0, 2 \rangle$

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 3 & 0 & 2 \end{vmatrix} = \langle -2, 4, 3 \rangle$$

$$0 = \langle -2, 4, 3 \rangle \cdot \langle x - 0, y - 1, z - 2 \rangle$$

$$-4 - 6 = -10$$

2. Identify the surface $2x^2 + 3z^2 = 4x + 2y^2$ through completing the square.

A. Cone

B. Ellipsoid

C. Parabolic hyperboloid

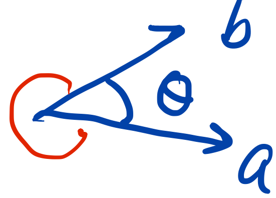
D. Hyperboloid of one sheet

E. Hyperboloid of two sheets

$$2(x^2 - 2x + 1) - 2y^2 + 3z^2 = 2$$

$$2(x-1)^2 - 2y^2 + 3z^2 = 2$$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$



$\theta \in [0, \pi]$

3. Find the angle θ between the velocity and acceleration at $t = 1$ for the position vector $\mathbf{r}(t) = \langle \cos t,\, t^2/2,\, -\sin t\rangle$.

A. $\theta = \pi/3$

B. $\theta = \pi/6$

C. $\theta = -\pi/6$

D. $\theta = -\pi/3$

E. $\theta = 5\pi/6$

?

$$\vec{v}(t) = \vec{r}\,'(t) = \langle -\sin t,\ t,\ -\cos t\rangle$$

$$\vec{a}(t) = \vec{v}\,'(t) = \langle -\cos t,\ 1,\ \sin t\rangle$$

$$\cos\theta = \frac{\vec{v}(t)\cdot\vec{a}(t)}{|\vec{v}(t)|\,|\vec{a}(t)|} = \frac{\sin t\cos t + t - \cos t\sin t}{\sqrt{1+t^2}\,\sqrt{2}}$$

$$= \frac{t}{\sqrt{2}\sqrt{1+t^2}}\Bigg|_{t=1} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

4. Compute the curvature of the curve $\mathbf{r}(t) = \langle 2\sin t, 1, 2\cos t\rangle$ at $t = \pi/4$.

A. $1/4$

B. 2

C. $1/2$

D. $\sqrt{2}$

E. $3/\sqrt{2}$

$$K = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|}$$

$$\vec{r}\,'(t) = \langle 2\cos t,\ 0,\ -2\sin t\rangle = 2\langle \cos t,\ 0,\ -\sin t\rangle$$

$$|\vec{r}\,'(t)| = |2|\,|\langle \cos t,\ 0,\ -\sin t\rangle| = 2\sqrt{\cos^2 t + \sin^2 t} = 2$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \langle \cos t,\ 0,\ -\sin t\rangle$$

$$\vec{T}\,'(t) = \langle -\sin t,\ 0,\ -\cos t\rangle$$

$$|\vec{T}\,'(t)| = 1$$

$$K = \frac{1}{2}$$

5. Find the length of the curve $\mathbf{r}(t) = \langle \sin t - t\cos t, \cos t + t\sin t\rangle$, $0 \le t \le 2\pi$.

A. π^2

B. $\pi^2/2$

C. $2\pi^2$

D. $4\pi^2$

E. 2π

$$L = \int_0^{2\pi} |\vec{r}\,'(t)|\,dt = \int_0^{2\pi} t\,dt = \frac{1}{2}t^2\Big|_0^{2\pi} = 2\pi^2$$

$$\vec{r}\,'(t) = \langle \cancel{\cos t} - \cancel{\cos t} + t\sin t, -\cancel{\sin t} + \cancel{\sin t} + t\cos t\rangle$$

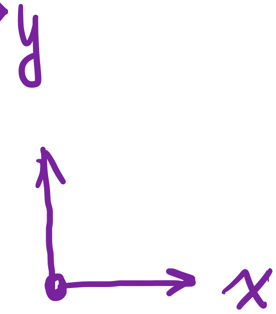
$$= \langle t\sin t, t\cos t\rangle = t\langle \sin t, \cos t\rangle$$

$$|\vec{r}\,'(t)| = |t|\,|\langle \sin t, \cos t\rangle| = t$$

6. A small metal ball is thrown vertically upward with a speed of $19.6\,m/s$, rises to a maximum height and then falls, eventually striking the ground. How high does the ball rise measured from its point of release? (Recall that the gravitational acceleration is $9.8\,m/s^2$.)

Given $\vec{v}(0) = \langle 0, 19.6\rangle$, $\vec{r}(0) = \langle 0, 0\rangle$

$\vec{a} = \langle 0, -g\rangle$



A. $16\,m$

B. $19.6\,m$

C. $9.8\,m$

D. $24\,m$

E. $12\,m$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle c_1, -gt + c_2\rangle$$

$$\langle 0, 19.6\rangle = \vec{v}(0) = \langle c_1, c_2\rangle$$

$$\Rightarrow \vec{v}(t) = \langle 0, -gt + 19.6\rangle$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 0, \underbrace{-\tfrac{1}{2}gt^2 + 19.6t}_{h(t)}\rangle$$

$$\vec{v}(t) = \langle 0, 0\rangle = \langle 0, -gt + 19.6\rangle$$

$$\Rightarrow t = \frac{19.6}{g} = 2, \quad h(2) = -\tfrac{1}{2}g \cdot 4 + 2 \times 19.6 = 19.6$$

7. Assuming that $\lim_{(x,y)\to(0,0)} \frac{\sin(4x^2+8y^2)}{x^2+2y^2}$ exists, what is its value?

$\to 0$ $\to 0$

$u = x^2+2y^2 \to 0$

A. 1/4

B. -4

C. $-1/4$

D. 0

E. 4

$$= \lim_{u\to 0} \frac{\sin(4u)}{u}$$

$$= \lim_{u\to 0} \frac{4\cos(4u)}{1} = 4 \lim_{u\to 0} \cos(4u) = 4$$

8. If $f(x,y) = \ln(x^2+y^4+2)$, compute $f_{xy}(2,1)$.

A. 4/7

B. $-4/7$

C. $-10/49$

D. $-16/49$

E. 12/49

$$f_x = \frac{1}{(x^2+y^4+2)} \cdot 2x$$

$$f_{xy} = 2x \cdot \frac{\partial}{\partial y}(x^2+y^4+2)^{-1} = 2x \cdot (-1)(x^2+y^4+2)^{-2} \cdot 4y^3$$

$$= -\frac{8xy^3}{(x^2+y^4+2)^2}\Bigg|_{(2,1)} = -\frac{8\cdot 2\cdot 1^3}{(4+1+2)^2}$$

$$= -\frac{16}{49}$$

9. Let $f(x,y) = xye^{xy}$, then the direction of steepest descent at $(2,3)$ is in the direction of the vector

A. $\langle -3, -2 \rangle$

B. $\langle 3, 2 \rangle$

C. $\langle 2, 3 \rangle$

D. $-\langle 2, 3 \rangle$

E. $\langle 1, -1 \rangle$

$$-\nabla f(2,3)$$

$$= -e^{xy}(1+xy)\langle y, x \rangle \Big|_{(2,3)}$$

$$= -c\,\langle 3, 2 \rangle$$

$$-\langle 3, 2 \rangle$$

$$f_x = y\left[e^{xy} + xe^{xy}\cdot y\right]$$

$$= ye^{xy}(1+xy)$$

$$f_y = xe^{xy}(1+xy)$$

10. Consider the function $f(x,y) = xy^4 - x - \frac{1}{2}x^2$ on $\mathbb{R}^2$. Among its critical points, this function has

A. an absolute maximum and an absolute minimum.

B. four critical points.

C. two local minima.

D. two saddle points.

E. a local maximum and a saddle point.

11. Consider the function $d(x,y) = \sqrt{(x-2)^2 + (y-2)^2 + 4}$ on the rectangular domain $[-2,2] \times [-1,1]$, that is, $-2 \leq x \leq 2$ and $-1 \leq y \leq 1$. On its domain:

A. it has a local maximum at $(0,0)$.

B. it has an absolute maximum value of 5 and an absolute minimum value of 2.

C. it has a local minimum with value 2.

D. it is a linear function.

E. it has an absolute minimum value of $\sqrt{5}$ and an absolute maximum value of $\sqrt{29}$.