eq. of a line L a pt $(x_0, y_0, z_0) \in L$, $\vec{v} = (v_1, v_2, v_3) \parallel L$

$$\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t\vec{v} = \langle x_0 + tv_1, y_0 + tv_2, z_0 + tv_3 \rangle$$

#1 l: $(2, 1, -3)$, $\vec{v} = \langle 2, -3, 7 \rangle$

$$\vec{r}(t) = \langle 2 + 2t, 1 - 3t, -3 + 7t \rangle$$

#2 $(1, -1, 0)$, $\vec{v} = (-2, 3, 5) - (1, -1, 0) = \langle -3, 4, 5 \rangle$

$$\vec{r}(t) = \langle 1 - 3t, -1 + 4t, 5t \rangle$$

eq. of a plane $\mathcal{P}$ $\quad (x_0, y_0, z_0) \in \mathcal{P}$, $\vec{n} = \langle n_1, n_2, n_3 \rangle \perp \mathcal{P}$

$$0 = \langle x - x_0, y - y_0, z - z_0 \rangle \cdot \vec{n}$$

$$\boxed{n_1 x + n_2 y + n_3 z = n_1 x_0 + n_2 y_0 + n_3 z_0}$$

#3 $\quad (1, -1, -1)$, $\vec{n} = \langle \frac{1}{2}, 2, 3 \rangle$

$$\frac{1}{2} x + 2y + 3z = \frac{1}{2} - 2 - 3 = -\frac{9}{2}$$

#4 $\quad (1, 0, -1)$

$\vec{a} = (-5, 3, 2) - (1, 0, -1) = \langle -6, 3, 3 \rangle \quad \vec{a} = \langle -2, 1, 1 \rangle$

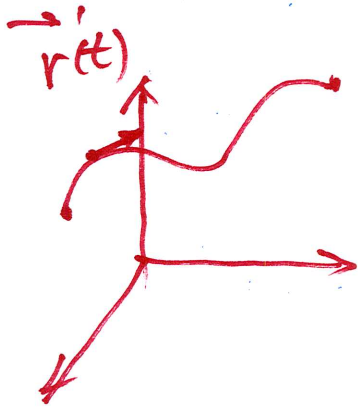
$\vec{b} = (2, -1, 4) - (1, 0, -1) = \langle 1, -1, 5 \rangle \quad \vec{b} = \langle 1, -1, 5 \rangle$

$\vec{a} \times \vec{b} = \langle 6, 11, 1 \rangle = \vec{n}$

$$6x + 11y + z = 6 - 1 = 5$$

$C: \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$a \le t \le b$



$$L = \int_C 1\, ds = \int_a^b \underbrace{|\vec{r}'(t)|\, dt}_{ds}$$

position $\vec{r}(t)$

velocity $\vec{v}(t) = \vec{r}'(t)$

acceleration $\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t)$

speed $s = |\vec{r}'(t)|$

#5 $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at $(2, 4, 8)$

$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle$

$(2, 4, 8) = \vec{r}(t) = \langle t, t^2, t^3 \rangle \Rightarrow t = 2$

$\vec{v} = \vec{r}'(2) = \langle 1, 4, 12 \rangle$

$\vec{r}_1(t) = \langle 2 + t, 4 + 4t, 8 + 12t \rangle$

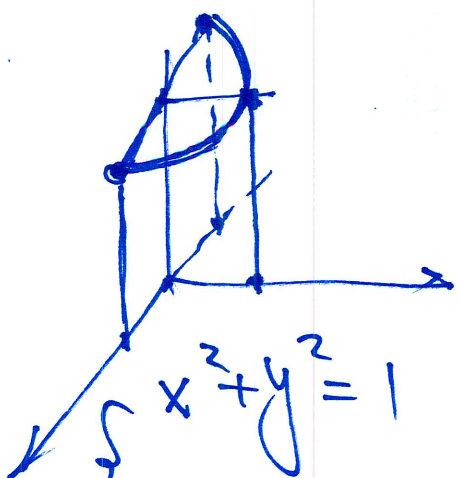
#6 $\vec{r}(t) = \langle \cos t, 3\sin t, -t^2 \rangle$, at $t = \pi$

$$\vec{v}(\pi) = \vec{r}\,'(\pi) = \langle -\sin t, 3\cos t, -2t \rangle \Big|_{t=\pi} = \langle 0, -3, -2\pi \rangle$$

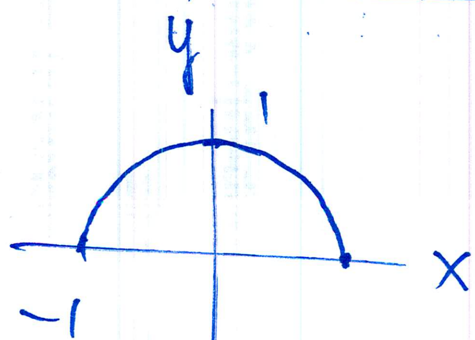
$$\vec{a}(\pi) = \vec{v}\,'(\pi) = \langle -\cos t, -3\sin t, -2 \rangle \Big|_{t=\pi} = \langle 1, 0, -2 \rangle$$

$$s\Big|_{t=4} = |\vec{v}(4)| = \sqrt{9 + 4\pi^2}$$

#7 $\underline{(1, 0, 5), (0, 1, 5), (-1, 0, 5)}$ semicircle



$$\begin{cases} x^2 + y^2 = 1 \\ z = 5 \end{cases}$$



$x^2 + y^2 = 1$

$\vec{r}(t) = \langle \cos t, \sin t, 5 \rangle$

$0 \le t \le \pi$

#8 $\vec{r}(t) = \langle \frac{2}{3}(1+t)^{\frac{3}{2}}, \frac{2}{3}(1-t)^{\frac{3}{2}}, t \rangle, -1 \le t \le 1$

$$L = \int_{-1}^{1} |\vec{r}\,'(t)|\,dt = \int_{-1}^{1} \sqrt{\left[(1+t)^{\frac{1}{2}}\right]^2 + \left[(1-t)^{\frac{1}{2}}\right]^2 + 1^2}\,dt$$

$$= \int_{-1}^{1} \sqrt{(1+t)+(1-t)+1}\,dt = \sqrt{3}\cdot 2$$

#9 $\boxed{f(x,y) = \sqrt{1-x^2-2y^2} = \text{const.}} = k$

$$1 - x^2 - 2y^2 = k^2 \Longrightarrow \boxed{x^2 + 2y^2 = 1 - k^2} \text{ ellipses}$$

#10 $f(x,y,z) = z - x^2 - y^2 = k$ — level surface

(1, 2, −3) : $k = z - x^2 - y^2 \Big|_{(1,2,-3)} = -3 - 1 - 4 = -8$

$$\begin{cases} \boxed{z - x^2 - y^2 = -8} \\ y = 0 \end{cases} \Longrightarrow \boxed{z = x^2 - 8}$$

#12 ~~$w(u,v)$, $u(t)$, $v(t)$~~

$w = f(u,v)$, $u = g_1(t)$, $v = g_2(t)$

$$\boxed{\frac{dw}{dt} = \frac{\partial w}{\partial u}\cdot\frac{du}{dt} + \frac{\partial w}{\partial v}\cdot\frac{dv}{dt}} = \frac{\partial w}{\partial u}\cdot g_1'(t) + \frac{\partial w}{\partial v}\cdot g_2'(t)$$

$$w = \frac{u^2}{v}, \quad \frac{dw}{dt} = \frac{2u}{v} g_1'(t) + (-1)\frac{u^2}{v^2} g_2'(t) = \frac{2g_1(t)}{g_2(t)} g_1'(t) - \frac{g_1(t)^2}{g_2(t)^2}\cdot g_2'(t)$$

$$\frac{dw}{dt}\Big|_{t=1} = \frac{2g_1(1)}{g_2(1)} g_1'(1) - \frac{g_1(1)^2}{g_2(1)^2} g_2'(1) = \frac{\cancel{2}\cdot 3}{\cancel{2}}\cdot 5 - \frac{3^2}{\cancel{2^2}}\cdot(\cancel{-4}) = 15 + 9 = 24$$

#13 $w = f(u,v)$, $u(r,s)$, $v(r,s)$

$= f(u(r,s), v(r,s))$

$w = e^{uv}$, $u = r+s$, $v = rs$

$$\boxed{\frac{\partial w}{\partial r} = \frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial r} + \frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial r}}$$

$$\frac{\partial w}{\partial r} = v e^{uv}\cdot 1 + u e^{uv}\cdot s = \left[rs + (r+s)s\right] e^{(r+s)rs}$$

$$= \left[2rs + s^2\right] e^{rs(r+s)}$$

#14 $f(x,y) = \cos(xy)$

$$\frac{\partial f}{\partial y} = -\sin(xy)\cdot x = -x\sin(xy)$$

$$\frac{\partial^2 f}{\partial x\,\partial y} = -\left[\sin(xy) + xy\cos(xy)\right]$$

#15 $\frac{\partial}{\partial x}\left[xy^2 + 3z(x,y) = \cos\left(z^2(x,y)\right)\right]$

$$LHS = y^2 + 3\frac{\partial z}{\partial x} = RHS = -\sin\left(z^2\right)\cdot \boxed{2z\cdot\frac{\partial z}{\partial x}}$$

$$\boxed{2z\cdot\frac{\partial z}{\partial x}} = \frac{\partial}{\partial x}\left(z^2\right)$$

$$\left[3 + 2z\sin z^2\right]\frac{\partial z}{\partial x} = -y^2$$

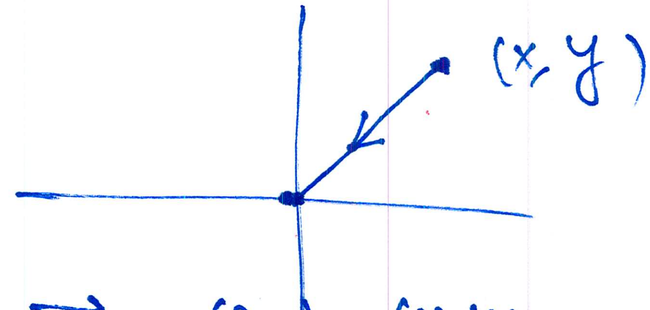
$$\Longrightarrow \frac{\partial z}{\partial x} = -\frac{y^2}{3+2z\sin z^2}$$

#16 $f(x,y) = xy^2$

$$\nabla f(2,3) = \cancel{\langle y^2} \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right\rangle\Big|_{(2,3)} = \left\langle y^2, 2xy\right\rangle\Big|_{(2,3)}$$

$$= \langle 9, 12\rangle$$

#17 $f(x,y) = 5 - 4x^2 - 3y$



$$D_{\vec{u}} f = \nabla f(x,y) \cdot \vec{u}$$

$$= \langle -8x, -3 \rangle \cdot \frac{-\langle x, y \rangle}{\sqrt{x^2+y^2}}$$

$$= \frac{8x^2+3y}{\sqrt{x^2+y^2}}$$

$$\vec{u} = \frac{(0,0)-(x,y)}{\sqrt{x^2+y^2}}$$

$$\vec{u} = \frac{-\langle x, y \rangle}{\sqrt{x^2+y^2}}$$

#18 $f(x,y) = x^2 y$, $\vec{u} = ? = \langle u_1, u_2 \rangle$

$$0 = D_{\vec{u}} f(2,3) = \nabla f(2,3) \cdot \vec{u} = \langle 2xy, x^2 \rangle \Big|_{(2,3)} \cdot \langle u_1, u_2 \rangle = \langle 12, 4 \rangle \cdot \langle u_1, u_2 \rangle$$

$$0 = 12u_1 + 4u_2 \Longrightarrow \boxed{3u_1 + u_2 = 0} \qquad u_2 = -3u_1$$

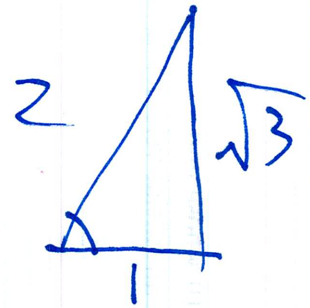
$\vec{u}$ is a unit vector $\Longleftrightarrow$ $u_1^2 + u_2^2 = 1$ $\qquad u_1^2 + 9u_1^2 = 1 \Rightarrow u_1^2 = \frac{1}{10}$

$$u_1 = \pm \frac{1}{\sqrt{10}}, \quad u_2 = \mp \frac{3}{\sqrt{10}} \qquad \frac{1}{\sqrt{10}} \langle 1, -3 \rangle \text{ or } \frac{1}{\sqrt{10}} \langle -1, 3 \rangle$$

#19 $f(x,y,z)=3xy-9xz^2+y$

$$\nabla f(1,1,0)=\langle 3y-9z^2,\ 3x+1,\ -18xz\rangle\Big|_{(1,1,0)}$$

$$=\langle 3,4,0\rangle$$



#20 $2\cos(xy\pi)=1$

$$\boxed{f(x,y)=\cos(xy\pi)=\frac{1}{2}}$$

$$\nabla f\left(\tfrac{1}{6},2\right)=\left\langle \frac{\partial f}{\partial x},\frac{\partial f}{\partial y}\right\rangle\left(\tfrac{1}{6},2\right)$$

$$=\langle -\sin(xy\pi)\cdot y\pi,\ -\sin(xy\pi)\ x\pi\rangle\Big|_{(\frac{1}{6},2)}$$

$$=-\left\langle \sin\frac{\pi}{3}\cdot 2\pi,\ \sin\left(\frac{\pi}{3}\right)\cdot\frac{\pi}{6}\right\rangle$$

$$=-\frac{\sqrt{3}}{2}\left\langle 2\pi,\ \frac{\pi}{6}\right\rangle$$

∇f — direction in which $f(x,y,z)$ increases most rapidly

- level surface $f(x,y,z)=C$

$$\vec{n}=\nabla f(x,y,z)$$

- $z=f(x,y)$

$$\vec{n}=\left\langle -\frac{\partial f}{\partial x},\ -\frac{\partial f}{\partial y},\ 1\right\rangle$$

- $\vec{r}(u,v)=\langle x(u,v),\ y(u,v),\ z(u,v)\rangle$

$$\vec{n}=\vec{r}_u\times\vec{r}_v$$