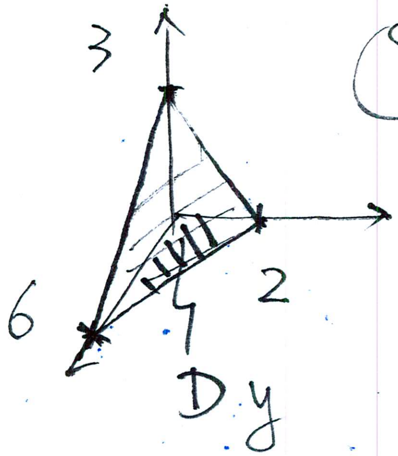
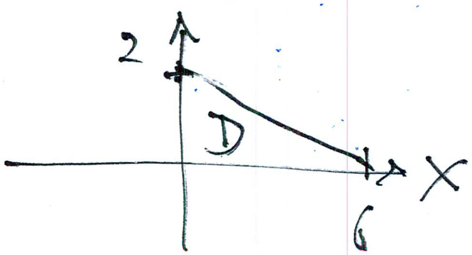
#32

$$x + 3y + 2z = 6 \Rightarrow \underline{z = \tfrac{1}{2}\left[6 - x - 3y\right]}$$



① $$A = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\,dy$$

$$= \iint_D \sqrt{1 + \left(-\tfrac{1}{2}\right)^2 + \left(\tfrac{-3}{2}\right)^2}\, dx\,dy$$



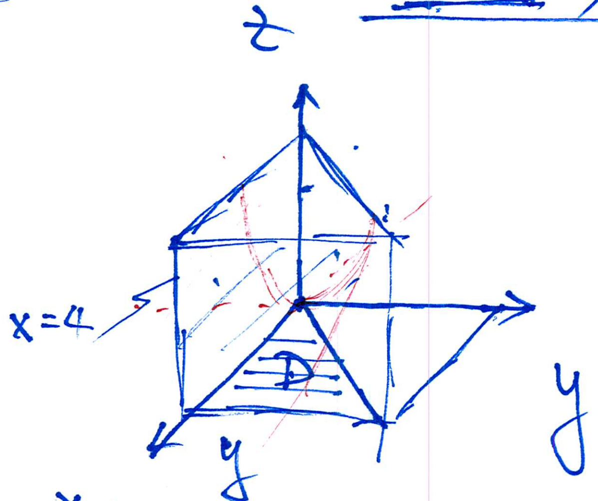
$$= \frac{\sqrt{14}}{2}\,|D| = \frac{\sqrt{14}}{2}\cdot\frac{1}{2}\cdot 2\cdot 6 = 3\sqrt{14}$$

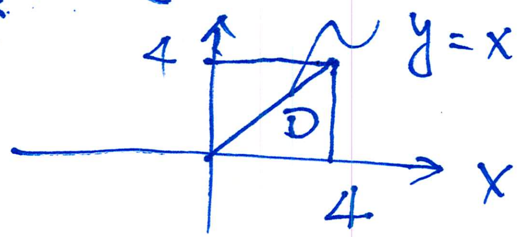
② $$\vec{r}(x,y) = \left\langle x,\ y,\ \tfrac{1}{2}\left[6 - x - 3y\right]\right\rangle,\quad (x,y)\in D$$

$$\vec{r}_x \times \vec{r}_y = \left\langle -\frac{\partial z}{\partial x},\ -\frac{\partial z}{\partial y},\ 1\right\rangle = \underline{\left\langle \tfrac{1}{2},\ \tfrac{3}{2},\ 1\right\rangle}$$

$$|\vec{r}_x \times \vec{r}_y| = \frac{\sqrt{14}}{2},\quad A = \iint_S dS = \iint_D |\vec{r}_x \times \vec{r}_y|\, dx\,dy$$

#33 : $z = y^2$, $y = x$, $y = 0$, $z = 0$





$$V = \iiint_E dV = \int_0^4 dx \int_0^x dy \int_0^{y^2} 1\, dz$$

$$E: \boxed{\begin{cases} 0 \le z \le y^2 \\ 0 \le y \le x \\ 0 \le x \le 4 \end{cases}}$$

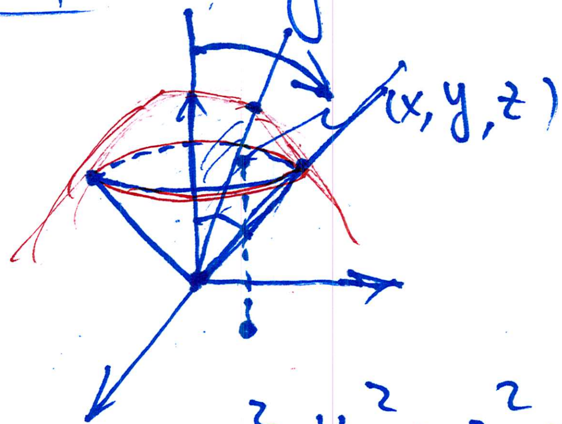
$$= \int_0^4 \int_0^x y^2\, dy\, dx$$

$$= \int_0^4 \frac{1}{3} y^3 \Big|_0^x dx$$

$$= \frac{1}{3} \int_0^4 x^3\, dx$$

$$= \frac{1}{3} \cdot \frac{1}{4} x^4 \Big|_0^4 = \frac{64}{3}$$

#34 $x^2+y^2+z^2=32$, $z^2=x^2+y^2$, $\rho(x,y,z)=z$



$$m=\iiint_E \rho\, dV=\int_{-4}^{4}\int_{-\sqrt{4^2-x^2}}^{\sqrt{4^2-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} \cancel{z\,dz}\ z\,dz\,dy\,dx$$

$$\begin{cases} x^2+y^2+z^2=32 \\ z^2=x^2+y^2 \end{cases} \Longrightarrow 2(x^2+y^2)=32$$

$$\Longrightarrow x^2+y^2=4^2$$

$$D=\{x^2+y^2\le 4^2\}$$

#35

$$\begin{cases} 0\le\rho\le\sqrt{32} \\ 0\le\theta\le 2\pi \\ 0\le\varphi\le\frac{\pi}{4} \end{cases}$$

$$\underset{\rho^2\cos^2\varphi}{\underset{\shortparallel}{z^2}}=\underset{\rho^2\sin^2\varphi}{\underset{\shortparallel}{x^2+y^2}} \Rightarrow \cos\varphi=\sin\varphi \Rightarrow \varphi=\frac{\pi}{4}$$

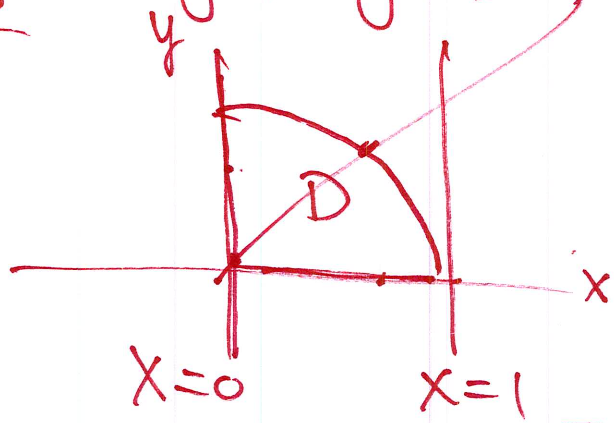
$$z\,dV=\rho\cos\varphi\ \rho^2\sin\varphi\ d\rho\,d\varphi\,d\theta$$

#36 $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 (x^2+y^2)^3 \, dy\, dx$

$y = \sqrt{1-x^2}$

$x^2+y^2 = 1$

$\begin{cases} 0 \le y \le \sqrt{1-x^2} \\ 0 \le x \le 1 \end{cases}$

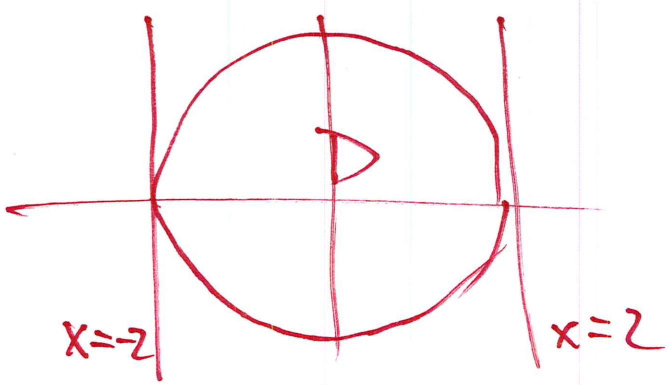


$= \int_0^{\frac{\pi}{2}} \int_0^1 r^2 \sin^2\theta \cdot r^6 \, r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \int_0^1 r^9 \sin^2\theta \, dr\,d\theta$

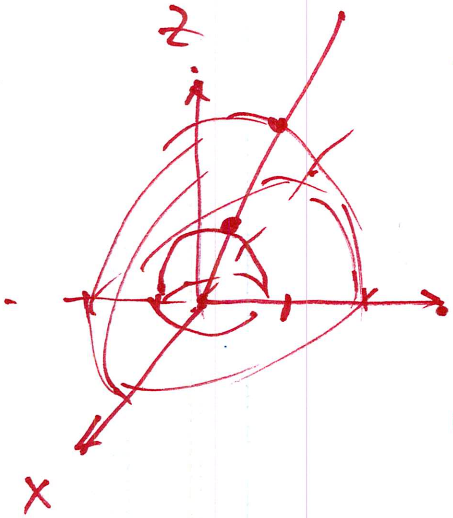
#37 $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} dz\, dy\, dx = \int_0^2 \int_0^{2\pi} \int_r^2 dz \; r\,d\theta\,dr$

$\boxed{\begin{aligned} dA &= dx\,dy \\ &= r\,dr\,d\theta \end{aligned}}$

$\begin{cases} -\sqrt{4-x^2} \le y \le \sqrt{4-x^2} \\ -2 \le x \le 2 \end{cases}$



\#38 $z = \sqrt{4-x^2-y^2}$, $z = \sqrt{1-x^2-y^2}$



$$\iiint_D \sqrt{x^2+y^2+z^2}\, dV$$

$$= \int_1^2 d\rho \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{2}} d\varphi\, \rho \cdot \rho^2 \sin\varphi$$

$$= 2\pi \left[\frac{1}{4}\rho^4\right]_1^2 \cdot \left[-\cos\varphi\right]_0^{\frac{\pi}{2}} = \frac{\pi}{2}\left[2^4 - 1\right] = \frac{15\pi}{2}$$

$$\boxed{\begin{aligned} dV &= dx\,dy\,dz \\ &= \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta \\ &= r\,dr\,d\theta\,dz \end{aligned}}$$

#39 $\vec{F}$ is conservative $\Longleftrightarrow$ $\vec{F} = \nabla f$ $\Longleftrightarrow$ $\boxed{\text{curl } \vec{F} = \vec{0}}$

✓ $\vec{F} = \langle \underbrace{xy^2 + x}_{P},\ \underbrace{x^2 y - y^2}_{Q} \rangle$

$$\nabla \times \vec{F} = \left\langle 0,\ 0,\ \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle = \langle 0,\ 0,\ 2xy - 2xy \rangle = \vec{0}$$

X $\vec{F} = \left\langle \frac{x}{y},\ \frac{y}{x} \right\rangle$

$$\nabla \times \vec{F} = \left\langle 0,\ 0,\ -\frac{y}{x^2} + \frac{x}{y^2} \right\rangle \neq \vec{0}$$

$\vec{F} = \langle ye^z,\ xe^z + e^y,\ (xy+1)e^z \rangle$

✓ $$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ ye^z & xe^z + e^y & (xy+1)e^z \end{vmatrix} = \langle xe^z - xe^z,\ ye^z - ye^z,\ e^z - e^z \rangle = \vec{0}$$

#40 A. $\underline{\underline{\text{curl}}}\ \nabla f = \vec{0}$ ✓, B. $\text{div}\ \underline{\underline{\text{curl}}}\ \vec{F} = 0$ ✓,

× C. $\nabla\left(\text{div}\ \vec{F}\right) \overset{?}{=} \vec{0}$ $\quad \vec{F} = \langle x^2, 1, 1\rangle$

$\text{div}\ \vec{F} = 2x + 0 + 0 = 2x$

$\nabla(2x) = \langle 2, 0, 0\rangle \neq \vec{0}$

D. $\text{curl}\ \vec{F} = \nabla \times \vec{F}$ ✓,

E. $\text{div}\ \vec{F} = \nabla \cdot \vec{F}$ ✓

#41 $m = \int_C \rho\, \underline{ds} = \int_0^2 \underset{\rho}{t} \cdot \underbrace{\left|\vec{r}\,'(t)\right| dt}_{ds} = \int_0^2 \underline{t}\sqrt{1+4t^2}\,\underline{dt}$

C: $y = x^2$, $x \in [0,2]$

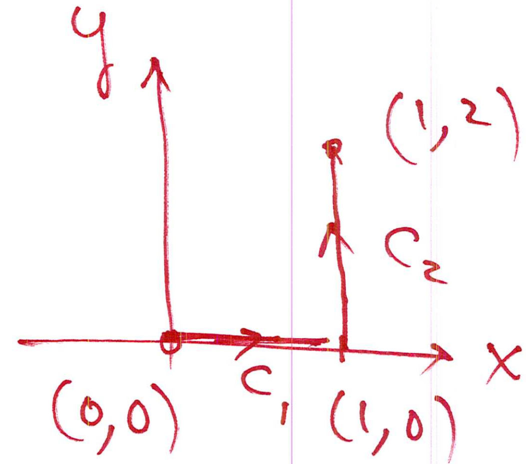
$\vec{r}(t) = \langle t, t^2\rangle$, $t \in [0,2]$

$\underline{\underline{u = 1+4t^2}}$
$du = 8t\,dt$

$\int_1^{17} \frac{1}{8} u^{\frac{1}{2}}\, du = \frac{1}{8}\cdot\frac{2}{3} u^{\frac{3}{2}}\Big|_1^{17}$

$= \frac{1}{12}\left[17\sqrt{17} - 1\right]$

\#42 $\int_C \vec{F} \cdot d\vec{r}$, $\vec{F} = \langle y, x^2 \rangle$,



$$= \int_C y\,dx + x^2\,dy$$

$$= \int_{C_1} y\,dx + x^2\,dy + \int_{C_2} y\,dx + x^2\,dy$$

$$= 0 + 0 + 0 + \int_0^2 1^2 \cdot dy = 2$$

#43

$$\int_C x\,dx + y\,dy + \cancel{z}^{\,xy}\,dz = \int_C \langle x, y, xy\rangle \cdot d\vec{r}$$

$$= \int_{-\frac{\pi}{2}}^{0} \langle \cos t, \sin t, \cos t \sin t\rangle \cdot \vec{r}\,'(t)\,dt$$

$$= \int_{-\frac{\pi}{2}}^{0} \langle \cos t, \sin t, \cos t \sin t\rangle \cdot \langle -\sin t, \cos t, -\sin t\rangle\,dt$$

$$= \int_{-\frac{\pi}{2}}^{0} -\cancel{2}\cos t \sin^2 t\,dt = -\int_{-\frac{\pi}{2}}^{0} \sin^2 t\,d\sin t$$

$$= -\frac{1}{3}\sin^3 t \Big|_{-\frac{\pi}{2}}^{0} = -\frac{1}{3}$$