Name: ______________

SI#: ______________

# Midterm 2– Math 262 (3/27/08)
## SHOW ALL RELEVANT WORK!!!

1. (10pts) Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & -1 \end{pmatrix}$, then $\det(2A)$ is

A. $-32$;

$$\det(A) = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 3 & -2 \end{vmatrix} = -4$$

B. $-16$;

$$\det(2A) = 2^3 \det(A) = 8 \cdot (-4) = -32$$

C. $-8$;

D. $-4$;

E. None of the above.

2. (10pts) Let $C_{ij}$ be the cofactor of the element $a_{ij}$ in the matrix $A = (a_{ij})_{3\times 3}$ with $\det(A) = 3$. The value of the expression $\underbrace{a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}}_{\det(A)} + \underbrace{a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}}_{\det(A)}$ then is equal to

A. 9;

B. 6;

$$2\det(A) = 6$$

C. 3;

D. 0;

E. None of the above.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 94 | 89 | 77 | 74 | 72 | 69 | 63 | 59 | 49 | 34 | |
| | 87 | 77 | 74 | 72 | 68 | 62 | 56 | 43 | | |
| | 82 | 76 | 73 | 72 | 66 | 60 | 55 | 42 | | |
| | 82 | 76 | 73 | 71 | 66 | | 54 | | | |
| | 80 | 76 | 73 | | 64 | | 52 | | | |
| 1 | 5 | | | 14 | | 8 | 5 | 3 | 1 | = 37 |

3. (10pts) If $A = \begin{pmatrix} 1 & 2 & 4 \\ 5 & 11 & 21 \\ 3 & 7 & 13 \end{pmatrix}$, determine a basis for row space and a basis for column space of $A$.

$$\begin{pmatrix} 1 & 2 & 4 \\ 5 & 11 & 21 \\ 3 & 7 & 13 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rowspace}(A) = \text{span}\{(1,2,4), (0,1,1)\}$$

$$\text{columnspace}(A) = \text{span}\left\{ \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 11 \\ 7 \end{pmatrix} \right\}$$

4. (10pts) Consider the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}$. The dimension of span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is then equal to

A. 1;

B. 2;

C. 3;

D. 4;

E. 5.

$$\begin{pmatrix} 1 & 1 & 0 & 2 \\ 2 & 1 & 3 & -1 \\ -1 & 1 & 1 & -2 \\ 2 & -1 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & -1 & 3 & -5 \\ 0 & 2 & 1 & 0 \\ 0 & -3 & 1 & -2 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & -1 & 3 & -5 \\ 0 & 0 & 7 & -10 \\ 0 & 0 & -8 & 13 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 7 & -10 \\ 0 & 0 & 0 & 13-10\times\frac{8}{7} \end{pmatrix}$$

5. (10pts) Let $S$ be the subspace of $R^3$ consisting of all vectors $\mathbf{x}$ of the form $\mathbf{x} = (r,\ r-2s,\ 3s-5r)$ with $r$ and $s$ being real numbers. A basis for $S$ is the pair

A. (1, −5, 1), (0, −2, 3);

$\vec{x} = r(1, 1, -5) + s(0, -2, 3)$

B. (1, 1, −5), (3, −2, 0);

C. (1, 1, −5), (−2, 0, 3);

D. (1, 1, −5), (0, −2, 3);

E. (−5, 1, 1), (3, −2, 0).

6. (10pts) Let $V = R^2$, and $S$ consists of all vectors $(x_1, x_2)$ satisfying $x_1^2 - x_2^2 = 0$. Express $S$ in set notation, and determine whether $S$ is a subspace of $V$.

$S = \{(x_1, x_2) \in R^2 \mid x_1^2 - x_2^2 = 0\}$

For any $(x_1, x_2)$, $(y_1, y_2) \in S$

$\Rightarrow x_1^2 - x_2^2 = 0$ and $y_1^2 - y_2^2 = 0$

But $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1,\ x_2 + y_2)$

and $(x_1+y_1)^2 - (x_2+y_2)^2$

$= (x_1^2 - x_2^2) + (y_1^2 - y_2^2) + 2(x_1y_1 - x_2y_2)$

$= 2(x_1y_1 - x_2y_2)$ may not be zero

$\Rightarrow$ not a subspace

7. (10pts)The vectors , (1, 2, 1), (3, 4, 5), and (2, 2, $k$) are linearly **dependent** if $k$ equals to

A. $-5$;

B. $-1$;

C. 0;

D. 4;

E. 8.

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 5 \\ 2 & 2 & k \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -2 & k-2 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & k-4 \end{pmatrix}$$

8. (10pts) Let $S = \{a_0 + a_1x + a_2x^2 + a_3x^3 \ : \ a_0,\ a_1,\ a_2,\ a_3 \in R\}$ be the set of all polynomials of degree less than or equal to 4. What is the dimension of $S$?

A. 1;

B. 2;

C. 3;

D. 4;

E. 5.

$S = \text{span}\{1, x, x^2, x^3\}$

9. (10pts) The **sum** of two eigenvalues of the matrix $A = \begin{pmatrix} 1 & 5 \\ 1 & -3 \end{pmatrix}$ is

A. $-8$;

B. $-4$;

C. $-2$;

D. $2$;

E. $4$.

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 5 \\ 1 & -3-\lambda \end{vmatrix}$$

$$= (\lambda-1)(\lambda+3) - 5 = \lambda^2 + 2\lambda - 8$$

$$= (\lambda-2)(\lambda+4)$$

$$\lambda_1 = -4, \ \lambda_2 = 2$$

$$\lambda_1 + \lambda_2 = -2$$

10. (10pts) Determine all eigenvalues and corresponding eigenvectors of $A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.

$$\det(A-\lambda I) = \begin{vmatrix} 5-\lambda & 0 & 0 \\ 0 & 5-\lambda & 0 \\ 0 & 0 & 5-\lambda \end{vmatrix} = (\lambda-5)^3$$

$$\lambda = 5$$

$$(A-5I)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Longrightarrow$ $x_1, x_2, x_3$ — arbitrary

eigenvectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$