Name: ____________________

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Midterm 2– Math 262 (3/27/08)
SHOW ALL RELEVANT WORK!!!

1. (10pts) Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 3 \end{pmatrix}$, then $\det(3A)$ is

A. 4;

$$\det(A) = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 3 & 2 \end{vmatrix} = 4$$

B. 12;

$$\det(3A) = 3^3 \cdot 4 = 27 \cdot 4 = 108$$

C. 36;

D. 108;

E. None of the above.

2. (10pts) Let C_{ij} be the cofactor of the element a_{ij} in the matrix $A = (a_{ij})_{2\times 2}$ with $\det(A) = 5$. The value of the expression $\underbrace{a_{11}C_{11}+a_{12}C_{12}}_{\det(A)}+\underbrace{a_{21}C_{21}+a_{22}C_{22}}_{\det(A)}$ then is equal to

A. 0;

B. 5;

C. 10;

D. 15;

E. None of the above.

94	89	77	74	72	69	63	59	49	34
	87	77	74	72	68	62	56	43	
	82	76	73	72	66	60	55	42	
	82	76	73	71	66		54		
	80	76	73		64		52		
1	5			14		8	5	3	1 = 37

3. (10pts) Let $V = R^2$, and S consists of all vectors (x_1, x_2) satisfying $x_1^2 - x_2^2 = 0$. Express S in set notation, and determine whether S is a subspace of V.

4 $S = \left\{ (x_1, x_2) \in R^2 \,\middle|\, x_1^2 - x_2^2 = 0 \right\}$

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1,\ x_2 + y_2)$$

$$(x_1+y_1)^2 - (x_2+y_2)^2 = (x_1^2 - x_2^2) + (y_1^2 - y_2^2) + 2(x_1y_1 - x_2y_2)$$

$$= 2(x_1y_1 - x_2y_2) \quad \text{may not be zero} \quad 6$$

it is not a subspace

spanning sets ~~basis functions~~ for

4. (10pts) If $A = \begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & -2 \\ -1 & 3 & -7 \end{pmatrix}$, determine both row and column spaces of A.

$$\begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & -2 \\ -1 & 3 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -6 \\ 0 & 1 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rowspace}(A) = \text{span}\{(1, -2, 1),\ (0, 1, -6)\}$$

$$\text{columnspace}(A) = \text{span}\left\{ \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ -7 \\ 3 \end{pmatrix} \right\}$$

5. (10pts) Let S be the subspace of R^3 consisting of all vectors $\mathbf{x}$ of the form $\mathbf{x} = (r+s,\ r-s,\ 2r+2s)$ with r and s being real numbers. A basis for S is the pair

A. (2, 0, 4), (2, 0, 2);

B. (1, 1, 2), (2, −1, 1);

C. (1, 1, 2), (−1, 1, 2);

D. (2, 1, 1), (2, −1, 1);

E. (1, 1, 2), (1, −1, 2).

$$\vec{x} = (r, r, 2r) + (s, -s, 2s)$$
$$= r(1, 1, 2) + s(1, -1, 2)$$

6. (10pts) Consider the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$. The dimension of span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is then equal to

A. 1;

B. 2;

C. 3;

D. 4;

E. 5.

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 3 & 1 & 7 & 3 \\ 5 & -3 & 9 & 1 \\ -2 & 4 & 2 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 4 & 4 & 6 \\ 0 & 2 & 4 & 6 \\ 0 & 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 4 & 4 & 6 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 2 & 2 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

7. (10pts) Let $S = \{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 \ : \ a_0,\ a_1,\ a_2,\ a_3,\ a_4 \in R\}$ be the set of all polynomials of degree less than or equal to 4. What is the dimension of S?

A. 1;

$$S = \text{span}\{1, x, x^2, x^3, x^4\}$$

B. 2;

C. 3;

D. 4;

E. 5.

8. (10pts) The vectors , (1, 2, 1), (3, 4, 5), and (2, −2, k) are linearly **dependent** if k equals to

A. −5;

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 5 \\ 2 & -2 & k \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -6 & k-2 \end{pmatrix}$$

B. −1;

C. 0;

D. 4;

$$\to \begin{pmatrix} 1 & 2 & 1 \\ 0 & +1 & -1 \\ 0 & 0 & k-8 \end{pmatrix}$$

E. 8.

9. (10pts) The **product** of two eigenvalues of the matrix $A = \begin{pmatrix} 1 & 6 \\ 2 & -3 \end{pmatrix}$ is

A. 5;

B. 3;

C. 1;

D. −5;

E. −15.

$$\det \begin{pmatrix} 1-\lambda & 6 \\ 2 & -3-\lambda \end{pmatrix} = (\lambda-1)(\lambda+3) - 12$$

$$= \lambda^2 + 2\lambda - 15$$

$$= (\lambda-3)(\lambda+5)$$

$$\lambda_1 = 3, \quad \lambda_2 = -5$$

10. (10pts) Determine all eigenvalues and corresponding eigenvectors of $A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

$$\det(A - \lambda I) = \det\begin{pmatrix} 3-\lambda & 0 \\ 0 & 3-\lambda \end{pmatrix} = (\lambda-3)^2$$

$$\lambda = 3$$

$$(A - 3I)\vec{v} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\vec{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$