a.
$$n = 0$$
: $\alpha = 0$

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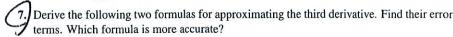
b.
$$n = 1$$
: $\alpha = 0$ and $\alpha = 1$

c.
$$n=2$$
: $\alpha=0$ and $\alpha=2$

- 5. Formula (9) for f''(x) is often used in the numerical solution of differential equations. By adding the Taylor series for f(x + h) and for f(x h), show that the error in this formula has the form $\sum_{n=1}^{\infty} a_{2n}h^{2n}$. Determine the coefficients a_{2n} explicitly. Also derive the error term given in Equation (9).
- **6.** Derive the following two formulas for approximating derivatives and show that they are both $\mathcal{O}(h^4)$ by establishing their error terms:

$$f'(x) \approx \frac{1}{12h} \left[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h) \right]$$

$$f''(x) \approx \frac{1}{12h^2} \left[-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h) \right]$$



$$f'''(x) \approx \frac{1}{h^3} [f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)]$$
$$f'''(x) \approx \frac{1}{2h^3} [f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

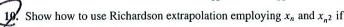
8. (Continuation) Carry out the instructions in the preceding problem for the following fourth-derivative formulas:

$$f^{(4)} \approx \frac{1}{h^4} \left[f(x+4h) - 4f(x+3h) + 6f(x+2h) - 4f(x+h) + f(x) \right]$$

$$f^{(4)} \approx \frac{1}{h^4} \left[f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h) + f(x-2h) \right]$$

9. Show that in Richardson extrapolation,

$$D(2,2) = \frac{16}{15}\psi(h/2) - \frac{1}{15}\psi(h)$$



$$L = x_n + a_1 n^{-1} + a_2 n^{-2} + a_3 n^{-3} + \cdots$$

11. Prove or disprove:

a. If
$$L - x_n = \mathcal{O}(n^{-1})$$
, then $L - (2x_{2n} - x_n) = \mathcal{O}(n^{-2})$.

b. If
$$L - x_n = \mathcal{O}(n^{-1})$$
, then $L - x_{n^2} = \mathcal{O}(n^{-2})$.

Discuss the numerical consequences of this problem.

12. Show how to use Richardson extrapolation if

$$L = \varphi(h) + a_1h + a_3h^3 + a_5h^5 + \cdots$$

- Suppose that $L = \lim_{h\to 0} f(h)$ and that $L f(h) = c_6 h^6 + c_9 h^9 + \cdots$. What combination of f(h) and f(h/2) should be the best estimate of L?
 - 14. Using Taylor series, derive the error term for the approximation

$$f'(x) \approx \frac{1}{2h} \left[-3f(x) + 4f(x+h) - f(x+2h) \right]$$