**6.1**

**9**. Let$F(x) = g(x) + \dfrac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$. We know that $g(x_0) = f(x_0)$, $h(x_n) = f(x_n)$, and $g(x_i) = h(x_i) = f(x_i)$ for $1 \le i \le n-1$. Hence

$F(x_0) = g(x_0) + \dfrac{x_0 - x_0}{x_n - x_0}[g(x_0) - h(x_0)] = g(x_0) + \dfrac{0}{x_n - x_0}[g(x_0) - h(x_0)] = g(x_0) = f(x_0)$,

$F(x_i) = g(x_i) + \dfrac{x_0 - x_i}{x_n - x_0}[g(x_i) - h(x_i)] = g(x_i) + \dfrac{x_0 - x_i}{x_n - x_0}[0] = g(x_i) = f(x_i)$, for $1 \le i \le n-1$ and

$F(x_n) = g(x_n) + \dfrac{x_0 - x_n}{x_n - x_0}[g(x_n) - h(x_n)] = g(x_n) - [g(x_n) - h(x_n)] = h(x_n) = f(x_n)$

**14**. $f^{(n)}(x) = \cosh x$ when $n$ is odd and $f^{(n)}(x) = \sinh x$ when $n$ is even. Let $M = Max(\sinh x, \cosh x)$ for $-1 \le x \le 1$. Then $M < 2$. Also $|x| \le |\sinh x| = f(x)$. By Theorem 2, we have

$|p(x) - f(x)| = |\dfrac{1}{n!} f^{(n)}(\xi_x) x \prod_{i=1}^{n-1}(x - x_i)| \le \dfrac{1}{n!} M |f(x)| 2^{n-1} \le \dfrac{2^n}{n!}|f(x)|$

**22**. Lagrange: $p(x) = 0 \cdot \dfrac{x(x-1)}{2} + 1 \cdot \dfrac{(x+2)(x-1)}{-2} + (-1) \cdot \dfrac{x(x+2)}{3} = -\dfrac{5x^2}{6} - \dfrac{7x}{6} + 1$,

Newton: $p(x) = 0 + \dfrac{1}{2}(x+2) - \dfrac{5}{6}x(x+2) = -\dfrac{5x^2}{6} - \dfrac{7x}{6} + 1$. They are same.

**27**. $f^{(n)}(x) = e^{x-1} = f(x)$ and $f(x) \le 1$ for $-1 \le x \le 1$. By Theorem 2, we have

$|f(x) - p(x)| = |\dfrac{1}{13!} f^{(13)}(\xi_x) \prod_{i=0}^{12}(x - x_i)| \le \dfrac{1}{13!} \cdot 1 \cdot 2^{13} = \dfrac{2^{13}}{13!}$

**6.2**

**5**. Plug $p(x)$ in $f(x)$ on equation (10) on page 329.

**6**. We will prove this by induction on the number of nodes $k$. For $k = 2$, $(\alpha f + \beta g)[x_0, x_1] = \dfrac{(\alpha f + \beta g)(x_1) - (\alpha f + \beta g)(x_0)}{x_1 - x_0} = \dfrac{\alpha f(x_1) - \alpha f(x_0) + \beta g(x_1) - \beta g(x_0)}{x_1 - x_0} = \alpha f[x_0, x_1] + \beta g[x_0, x_1]$.

Assume that this is true for $k = n - 1$. When $k = n$, we have

$(\alpha f + \beta g)[x_0, x_1, ..., x_n] = \dfrac{(\alpha f + \beta g)[x_1, ..., x_n] - (\alpha f + \beta g)[x_0, ...x_{n-1}]}{x_n - x_0}$

$= \dfrac{\alpha f[x_1 ..., x_n] + \beta g[x_1 ..., x_n] - \alpha f[x_0, ...x_{n-1}] - \beta g[x_0, ...x_{n-1}]}{x_n - x_0}$

$= \dfrac{\alpha f[x_1 ..., x_n] - \alpha f[x_0, ...x_{n-1}] + \beta g[x_1 ..., x_n] - \beta g[x_0, ...x_{n-1}]}{x_n - x_0}$

$= \alpha f[x_0 ..., x_n] + \beta g[x_0 ..., x_n]$

**17**. $p(x) = 3 + \dfrac{1}{2}(x-1) + \dfrac{1}{3}(x-1)(x - \dfrac{3}{2}) - 2(x-1)(x - \dfrac{3}{2})x$

**19**. Let$F(x) = \dfrac{(x_n - x)u(x) + (x - x_0)v(x)}{x_n - x_0}$. We know that $u(x_0) = f(x_0)$, $v(x_n) = f(x_n)$, and $u(x_i) = v(x_i) = f(x_i)$ for $1 \le i \le n-1$. Hence

$F(x_0) = \dfrac{(x_n - x)u(x) + (x - x_0)v(x)}{x_n - x_0} = \dfrac{(x_n - x_0)u(x_0) + 0}{x_n - x_0} = u(x_0) = f(x_0)$,

$F(x_i) = \dfrac{(x_n - x_i)u(x_i) + (x_i - x_0)v(x_i)}{x_n - x_0} = \dfrac{(x_n - x_i)f(x_i) + (x_i - x_0)f(x_i)}{x_n - x_0} = \dfrac{(x_n - x_0)f(x_i)}{x_n - x_0} = f(x_i)$,

for $1 \le i \le n-1$ and

$F(x_n) = \dfrac{(x_n - x_n)u(x_n) + (x_n - x_0)v(x_n)}{x_n - x_0} = \dfrac{0 + (x_n - x_0)v(x_n)}{x_n - x_0} = v(x_n) = f(x_n)$