2.2 34

$$\frac{dT}{dt} = k(M-T)$$ separable.

$$\frac{1}{M-T}\,dT = k\,dt.$$

$$-\ln|M-T| = kt + c$$

$$\ln(M-T) = -kt + c$$

$$M-T = e^{-kt}\cdot C$$

$$\boxed{T = M - e^{-kt}\cdot C}$$

Plug in: $M = 70.$

$T(0) = 100$

$T(6) = 80$

$\Rightarrow T = 70 + 30e^{-kt}$

where

$e^{-k} = \left(\frac{1}{3}\right)^{1/6}.$

2.3

25(9)

$$y' + 2xy = 1$$

$$\mu = e^{\int 2x\,dx} = e^{x^2}$$

$$\Rightarrow e^{x^2}y' + e^{x^2}2xy = e^{x^2}$$

$$\left(e^{x^2}y\right)' = e^{x^2}$$

$$\Rightarrow e^{x^2}y = \int e^{x^2} + c$$

$$y = e^{-x^2}\cdot\int e^{x^2} + ce^{-x^2}$$

Now plug in $y(2) = 1$.

15/ $\underbrace{\cos\theta}_{M}\,dr - \underbrace{(r\sin\theta - e^{\theta})}_{N}\,d\theta = 0$

$M_\theta = -\sin\theta \qquad N_r = -\sin\theta \quad \Rightarrow$ exact

$\Downarrow$

$F = r\cos\theta + c(\theta) \qquad F = r\cos\theta + e^{\theta} + c(r)$

$F = r\cos\theta + e^{\theta} + c.$

扫描全能王 创建

2.6

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Let $x = u + h$ (like P79).

$y = v + l$.

$$\frac{dy}{dx} = \frac{dv}{du} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2} = \frac{a_1(u+h) + b_1(v+l) + c_1}{a_2(u+h) + b_2(v+l) + c_2}$$

Need to choose h, l. s.t.

$$\begin{cases} a_1 h + b_1 l + c_1 = 0 \\ a_2 h + b_2 l + c_2 = 0. \end{cases} > 2 \text{ var. } 2 \text{ eq}^n.$$

$\Rightarrow$ There is unique soln. of h, l.

So $\frac{dy}{dx} = \cancel{\frac{a_1 x + b_1 y}{}} \frac{dv}{du} = \frac{a_1 u + b_1 v}{a_2 u + b_2 v}$

$$= \frac{a_1 u + b_1 v}{a_1 k u + b_1 k v} = \frac{1}{k}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{k}$$