7.1 (28)

$$\begin{cases} ax + by = f \\ cx + dy = g \end{cases} \xrightarrow{REF} \left[\begin{array}{cc|c} a & b & f \\ c & d & g \end{array}\right]$$

①×($-\frac{c}{a}$)+②

$$\rightarrow \left[\begin{array}{cc|c} a & b & f \\ 0 & d-\frac{bc}{a} & g-\frac{fc}{a} \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & b/a & f/a \\ 0 & 1 & \frac{ag-fc}{ad-bc} \end{array}\right]$$

consistent => $\boxed{ad-bc \neq 0}$

7.2 (23)

$$\left[\begin{array}{ccccc} x & \cdot & \cdot & \cdot & \cdot \\ 0 & x & \cdot & \cdot & \cdot \\ 0 & 0 & x & \cdot & \cdot \end{array}\right]_{3\times5}$$

consistent

as x_4. x_5 are free variables.

x_1, x_2, x_3 can be expressed using x_4. x_5.

(31). Yes

eg. $\begin{cases} x+y=1 \\ 2x+2y=2 \\ 3x+3y=3 \end{cases}$ (This system actually has only 1 eqn)

7.4 (36)

$$A = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}_{5\times 3}$$

$$A_{5\times3}\, y_{3\times1} = z_{5\times1}$$

$Ay = z$ is consistent $\Rightarrow$ $Ax = 4z$ is consistent.

7.5 (38)

1. $\vec{y} = \vec{0}$. i.e. homogeneous.

$\Rightarrow$ $Ax = y$ has no free variable.

REF is $\begin{bmatrix} 1 & x & x \\ 0 & 1 & x \\ 0 & 0 & 1 \end{bmatrix}$. then. $Ax = z$ should always have unique solution.

2. $y \neq 0$. i.e. non-homogeneous.

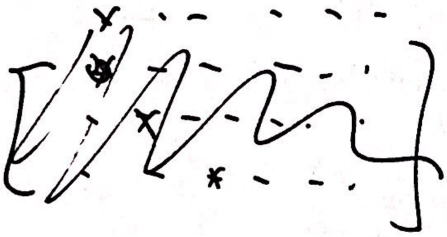
if $Ax = y$ has free variable, then $Ax = y$ has no solution. but setting the free variable and z_i to be zero. would make it has solution.

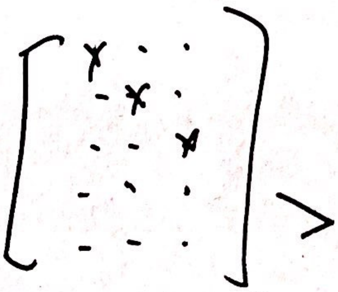
eg $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $y = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ (no) $z = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (yes).

7.7 (14).

$$\begin{bmatrix}1\\1\\h\end{bmatrix} = x\begin{bmatrix}1\\-1\\3\end{bmatrix} + y\begin{bmatrix}-5\\7\\8\end{bmatrix}$$

$\Rightarrow$ solve x. y. h.

(40).  n pivot columns.

 → all zeros. → these equations are linear combination of upper rows.

so. only zero or unique solution.