

$$9/ \quad 3x_1 + 4x_2 + x_3 - x_4 = 0$$

1 eqn. ... 3 free var.

$$u_1 = \begin{pmatrix} -4/3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} -1/3 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$u_3 = \begin{pmatrix} 1/3 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

So $\{u_1, u_2, u_3\}$ are a basis of W .

$$2/. \quad T(2v_1 + 3v_2) = v_1 + v_2 \quad (*)$$

$$T(2v_1 + v_2) = 3v_1 - v_2$$

What is $T(-2v_1 + v_2)$.

Note. $-2v_1 + v_2$ could be written as a linear combination of $2v_1 + 3v_2$ & $2v_1 + v_2$

$$\begin{aligned} \therefore -2v_1 + v_2 &= a(2v_1 + 3v_2) + b(2v_1 + v_2) \\ &= \underline{2a}v_1 + 3av_2 + \underline{2b}v_1 + bv_2 \\ &= (2a + 2b)v_1 + (3a + b)v_2 \end{aligned}$$

Equate coefficients:

$$\left. \begin{aligned} -2 &= 2a + 2b \\ 1 &= 3a + b \end{aligned} \right\} \Rightarrow \begin{aligned} a &= 1 \\ b &= -2 \end{aligned}$$

$$\text{So } T(-2v_1 + v_2) = T(1(2v_1 + 3v_2) - 2(2v_1 + v_2))$$

$$\text{by } (*) \quad 1 \cdot (v_1 + v_2) - 2(3v_1 - v_2)$$

$$= -5v_1 + 3v_2 \quad //$$