

P149/11

$$\begin{bmatrix} 2 & 1 & 4 \\ 2 & -3 & 4 \\ 3 & -2 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -2 & 6 \\ 2 & -3 & 4 \\ 2 & 1 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & -3 & 4 \\ 2 & 1 & 4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -5 & 0 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow \underline{\text{Rank 2}}$$

$$13/. \begin{bmatrix} 2 & -1 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 7 \end{bmatrix}$$

$\Rightarrow$  Rank 2.

$$23/. \left[ \begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 3 & 5 & -4 & 16 \\ 2 & 3 & -a & b \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 4-a & b-8 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 3-a & b-10 \end{array} \right]$$

(a) No solution:  $a=3$   $b \neq 10$

(b) Infinitely many:  $a=3$   $b=10$

(c) Unique.  $a \neq 3$ .

$$26/ \left[ \begin{array}{cc|c} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cc|c} 1 & a_{12}/a_{11} & b_1/a_{11} \\ 0 & \Delta/a_{11} & \Delta_2/a_{11} \end{array} \right]$$

(a), If  $\Delta \neq 0$ .  $\text{rank } A = 2$ .

$$x_1 = \frac{\Delta_1}{\Delta} \quad x_2 = \frac{\Delta_2}{\Delta}$$

(b),  $\left[ \begin{array}{cc|c} 1 & a_{12}/a_{11} & \frac{b_1}{a_{11}} \\ 0 & 0 & \Delta_2 \end{array} \right]$  (i), No sol if  $\Delta_2 \neq 0$ .

(ii),  $\Delta_2 = 0$   $\text{rank } A \neq 2$ .

(c), Unique sol. Intersecting.

No sol: parallel lines

$\infty$  sol: two lines on top of each other.

$$47/ \left[ \begin{array}{ccc} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 1 & 1 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{ccc} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{array} \right]$$

$\Rightarrow$  Only zero (trivial) solution  $\vec{x} = \vec{0}$ .