Quiz 1

1. Determine all values of the constant \( r \) such that the given function solves the following ODE
   \[ y = x^{2r}, \quad x^2 y'' + xy' - y = 0 \]

2. Solve this ODE
   \[ \frac{dy}{dx} = \frac{x^2 + 1}{y^3} \]
   with \( y(0) = 2 \)

Quiz 2

1. Find the general solution of \( xy' - y = x^2 e^x \)

2. If \( y' = \frac{3(x+1)^2}{y} \), and \( y(-1) = 2 \). What is \( y(0) \) ?

Please EXAMINE your answer by substitute your solution \( y \) back to original ODE and make sure your \( y \) satisfies initial condition. (Thanks)

Quiz 3

1. Find the solution of \((3x^2 + y)dx + (x + 2y)dy = 0\) passing through \((1,1)\).

2. Find the general solution of the following Bernoulli equation:
   \[ y' + \frac{1}{x}y = x^2 y^2 \]

Quiz 4

1. Solve
   \[ (2x - y^2) \, dx + 2xy \, dy = 0 \]

2. Let
   \[ A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 1 & -1 & 2 \end{pmatrix} \]
   Compute the multiplication \( AA^T \).
Quiz 5

1. Find general solution of:

\[ y \, dx + (e^{-y} + x)\, dx = 0 \]

2. Using elementary row operations to transform \( A \) into REF. Please write out each step clearly.

\[
A = \begin{pmatrix} 3 & 5 & 3 \\ 2 & 3 & -1 \\ -2 & -1 & 1 \end{pmatrix}
\]

Bonus: If \( \mathbf{b} = (11, 4, -2)^T \), solve \( \mathbf{x} \) for \( A\mathbf{x} = \mathbf{b} \)

Quiz 6

Let

\[
A = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -3 & 0 \\ 0 & -1 & 1 \end{pmatrix}
\]

Calculate
1. \( det(A) \)
2. \( A^{-1} \)

Quiz 7

1. Consider the vectors \( v_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \), \( v_2 = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix} \), \( v_3 = \begin{pmatrix} 5 \\ -3 \\ 9 \end{pmatrix} \), \( v_4 = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix} \).

What is the dimension of the space span\{\( v_1, v_2, v_3, v_4 \)\}?

2. Calculate the Wronskian of the functions \( \{x, \sin x, \cos x\} \).

Quiz 8

(Final 2001)

1. Determine all the values for \( k \) such that the vectors \( (1, -1, 0), (1, 2, 2), (0, 3, k) \) are a basis for \( \mathbb{R}^3 \).
2. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T([1, 1]^T) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T([1, 0]^T) = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$. What is $T([1, 2]^T)$?