Quiz 4, MA341 Name ______________

Let $f: \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and let S be a dense subset of $\mathbb{R}$ such that $f(x) = 0$ for all $x \in S$. Show that $f(x) = 0$ for all $x \in \mathbb{R}$. (Assume by contradiction that $f(c) \neq 0$ for some $c \notin S$.)

Pf. Since f is continuous at c and $f(c) \neq 0$, there is a $\delta > 0$ so that if $|x - c| < \delta$, then $|f(x)| > \frac{|f(c)|}{2}$.

Since S is dense, there is an $x' \in S$ satisfying $0 < |x' - c| < \delta$, so that $f(x') = 0$. This contradiction $\Rightarrow$ $f(c) = 0$.