Quiz 3 Math 341   **Name** _____

Consider $\lim \frac{2n+(-1)^n}{3n^2+\sqrt{n}}$.

If this limit diverges, explain why. If this limit converges, compute the limit and justify your answer.

To receive credit, you must fully justify your answer.

Solution

First we write

$$\frac{2n+(-1)^n}{3n^2+\sqrt{n}} = \frac{n}{n^2} \cdot \frac{2+\frac{(-1)^n}{n}}{3+\frac{1}{n\sqrt{n}}}$$

By the Squeeze Theorem, $\lim \frac{(-1)^n}{n} = 0$, so the addition rule yields

$$\lim(2 + \frac{(-1)^n}{n}) = 2 + 0 = 2$$

The product rule yields that $\lim \frac{1}{n\sqrt{n}} = 0$

Hence the addition rule implies $\lim(3 + \frac{1}{n\sqrt{n}}) = 3 + 0 = 3$

From the quotient rule, we obtain

$$\lim \frac{2+\frac{(-1)^n}{n}}{3+\frac{1}{n\sqrt{n}}} = \frac{2}{3}$$

Finally the product rule implies that

$$\lim \frac{1}{n} \cdot \left(\frac{2+\frac{(-1)^n}{n}}{3+\frac{1}{n\sqrt{n}}} \right) = 0 \cdot \frac{2}{3} = 0$$