Exam 1 Math 341 **Name** ____________________

1. Evaluate $\lim \frac{2n+3}{\sqrt{3n^2+2}}$.

2. Suppose $\lim(x_n) = 0$ and (y_n) is a bounded sequence with no limit.

(a) Find $\lim(x_n y_n)$. Prove that your answer is correct.

(b) Evaluate $\lim \frac{2x_n+3}{2+x_n y_n}$. Prove your answer.

3. Suppose $\lim(z_n) = z$ and $z \neq 0$.

   Show that if n is sufficiently large, then $|z_n| > \frac{|z|}{2}$.

4. Using problem 3, show that $\lim \frac{1}{z_n} = \frac{1}{z}$.

5. Suppose that a sequence (x_n) is defined by $x_{n+1} = \frac{2}{5}x_n + \frac{3}{2}, x_1 = 1$

(a) Find a positive number p so that if $x_n < p$ then $x_{n+1} < p$

(b) Show by induction that for all n, $x_n < x_{n+1}$

(c) Why does $\lim(x_n)$ exist?

(d) What is $\lim(x_n)$?

6. (a) State the Bolzano-Weierstrass Theorem.

(b) Give the definition of an "upper bound" of a set S.

(c) Give the definition of the supremum of a set S.

7. (a) State the Ratio Test for a sequence (x_n).

(b) Use the Ratio Test to compute $\lim(\frac{2n-1}{3^n})$.

8. (a) Find a bijection of N onto Z.

(b) Using the standard "diangonal" argument which shows that the set of positive rational numbers is denumerable, what is the 7-th rational number?