

# List of Topics for Exam 1, Math 341.

Assume that  $\lim x_n = x$  and

$$\lim y_n = y.$$

You should know the proofs of the following results:

①  $\lim (x_n + y_n) = x + y$  p. 64

②  $|x_n|$  is bounded, i.e., there is an  $M > 0$  so that  $|x_n| \leq M$  for all  $n$ . p. 63

③ Use the result in #2 to prove that

$$\lim x_n y_n = xy$$
 p. 64

④ If  $y \neq 0$ , then there is a constant  $K$

so that if  $n \geq K$ , then  $|y_n| > \frac{|y|}{2}$  p. 65

5 Use the result in # 4 to show that

if  $y \neq 0$ , then  $\lim (1/y_n) = 1/y$ . p. 65

6 If  $(x_n)$  tends to  $+\infty$  and if p. 92

$\lim y_n = y \neq 0$ , then  $\lim (x_n y_n) = +\infty$

7 If  $\lim x_n = 0$  and  $|y_n| \leq M > 0$  for

all  $n \in \mathbb{N}$ , then  $\lim (x_n y_n) = 0$  p. 64

8 State the definition of  $\sup S = u$ .

Show that if  $\sup S = u$  and

$w < u$ , then there is an element  $s_0 \in S$

such that  $w < s_0 \leq u$ . p. 37-38.

(12)

The polynomial equation

$$x^3 - 5x + 1 = 0 \text{ has a root } \pi$$

with  $0 < \pi < 1/2$ . Find a

contractive sequence that can be

used to calculate  $\pi$ . What is

the constant of the sequence p. 88-90

(13)

Prove Bernoulli's Inequality

(by induction), p. 30

(14)

Use the standard "diagonal" argument,

which shows that the positive rational

numbers are denumerable, what is

the 11-th prime.

15) Use the Ratio Test, show that p. 69

$$\lim n^2 b^n = 0 \text{ if } 0 < b < 1.$$

16) Compute  $\lim \frac{n!}{n^n}$

17) State the Bolzano - Weierstrass Thm.

p. 81.