

Ex. 1 Sketch the surface

$$x^2 + 4y^2 = 4$$

horizontal trace ($z = c$)

is an ellipse, independent
of z .



Ex. 2 Sketch
the surface

$$z = -y^2$$



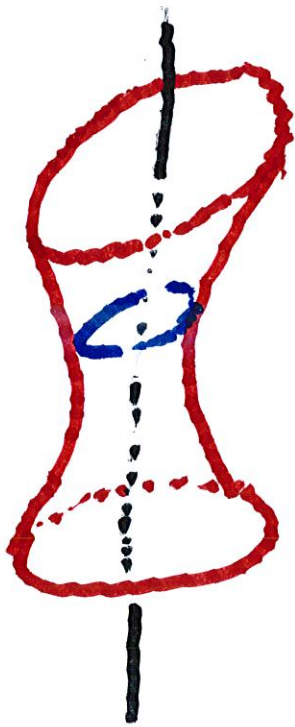
a parabolic
cylinder

Ex. 3

2

Sketch $x^2 + y^2 - 4z^2 = 4$

For each z , the horizontal trace is a cylinder of radius of radius $\sqrt{4+4z^2}$



Each vertical trace is a hyperbola

$$(y = c)$$

$$x^2 - 4z^2 = 4 - c^2$$

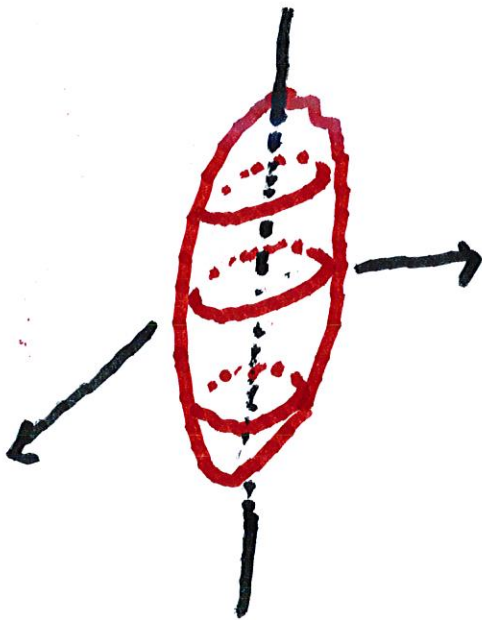
Ex. 4. Sketch the surface

$$x^2 + 4z^2 + y^2 = 4$$

Look at the horizontal

trace $x^2 + y^2 = 4 - 4z^2$

This is a circle if $z^2 < 1$



This is an
ellipsoid.

Ex. 5 Sketch

$$x^2 - y^2 + 4z^2 = -16$$

If we look at the y -trace,

we get $x^2 + 4z^2 = y^2 - 16$.

If $y^2 < 16$, then there is

NO SOLUTION. But if $y^2 > 16$,

the trace
is an ellipse



This is
a hyperboloid of 2 sheets

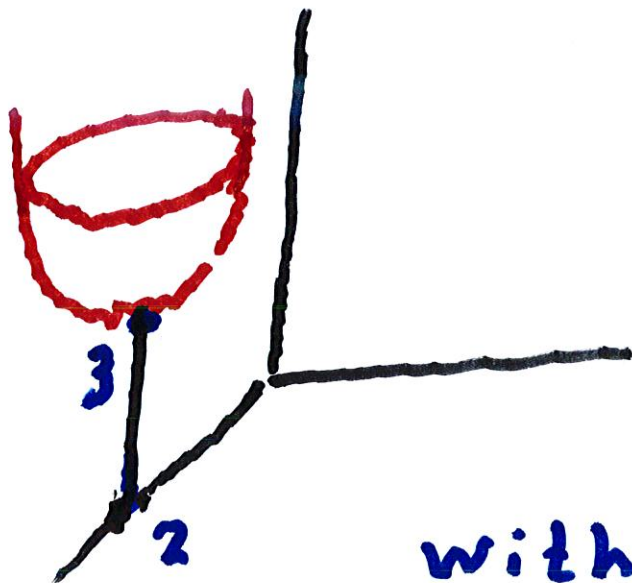
Ex. 6. Sketch

$$z - x^2 - y^2 + 4x - 7 = 0$$

$$z = x^2 - 4x + y^2 + 7$$

$$z = (x-2)^2 + y^2 + 7 - 4$$

$$z = (x-2)^2 + y^2 + 3$$



This is a
paraboloid

with vertex

at $(2, 0, 3)$

Ex. Find the equation
 for the points that
 are equidistant from

$(-1, 0, 0)$ and the plane $x = 1$
 d_1 d_2

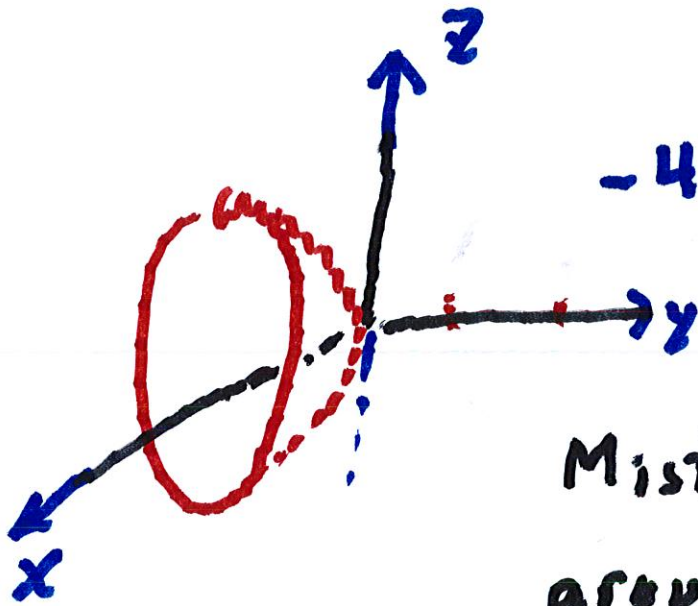
$$\sqrt{(x+1)^2 + y^2 + z^2} = 1 + x$$

$$(x+1)^2 + y^2 + z^2 = 1 + 2x + x^2$$

$$\rightarrow y^2 + z^2 = -4x$$

A paraboloid
 with axis =
 neg x axis





$$-4x = y^2 + z^2$$

Mistake: Should go around neg. $-x$ -axis

Ex Sketch

$$x^2 - y^2 + z^2 - 4x - 2y - 2z = -8$$

$$(x-2)^2 - (y+1)^2 + (z-1)^2$$

$$= -8 + 4 - 1 + 1$$

$$= -4$$

$$x^2 + z^2 = -4 + y^2 \quad \text{If } |y| < 2$$



y -trace is a circle.

Note $\sqrt{(x+1)^2 + y^2 + z^2}$

= dist. of (x, y, z) to $(-1, 0, 0)$

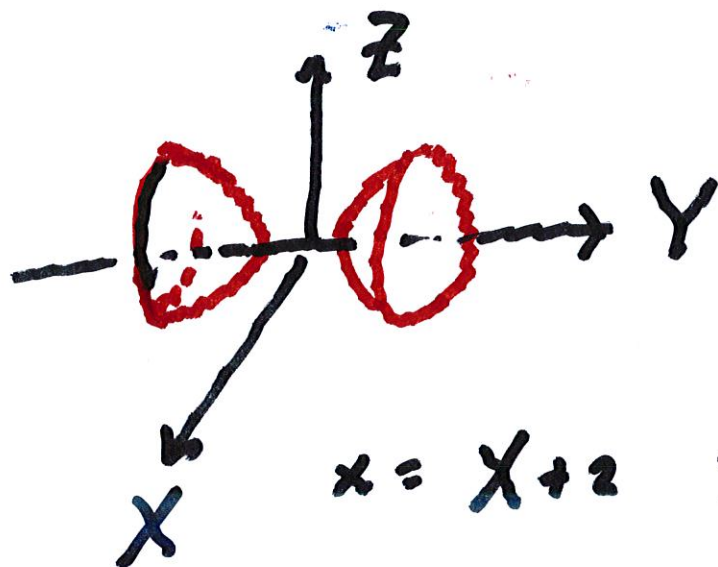
and $1-x$ = dist. of (x, y, z)

to plane $x = 1$.

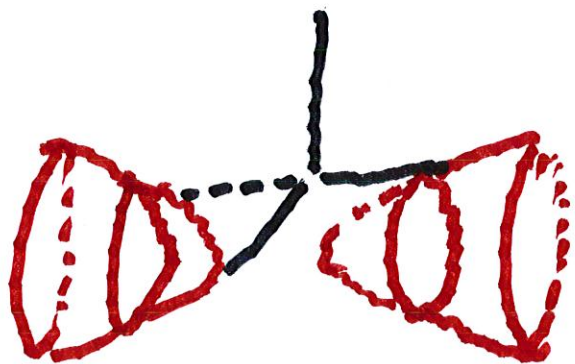
$$\therefore (x-2)^2 + (z-1)^2 = (y+1)^2 - 4$$

$$\text{Set } X = x-2, \quad Y = y+1, \quad Z = z-1$$

$$\rightarrow X^2 + Z^2 = Y^2 - 4$$



$$x = X+2 \quad y = Y-1 \quad z = Z-1$$



Center at

$$(2, -1, -1)$$

Ex. 1. Find parametric equations of line that lies intersection of

$$x + y + z = 1 \quad x + 2y + 3z = 2$$

 P_1
 P_2

$$\vec{n}_1 = \langle 1, 1, 1 \rangle$$

$$\vec{n}_2 = \langle 1, 2, 3 \rangle$$

If \vec{v} = direction of line L , then

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \vec{i} - 2\vec{j} + \vec{k}$$

Must find a point in L

Set $z = 0$

$$\left. \begin{array}{l} x+y=1 \\ x+2y=2 \end{array} \right\} y=1, x=0$$

$\rightarrow (0, 1, 0)$ is in L .

$$\therefore \langle x, y, z \rangle = (0, 1, 0) + t \langle 1, -2, 1 \rangle$$

$$\rightarrow x = t, y = 1 - 2t, z = t$$

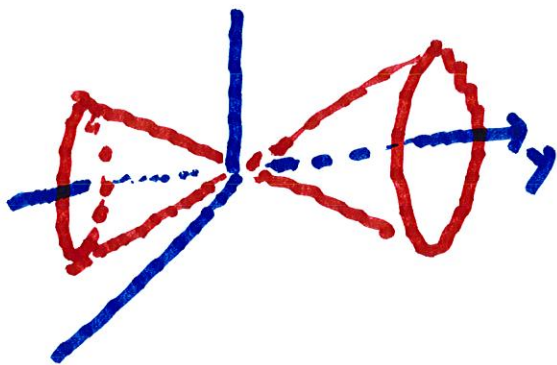
Ex. Sketch

$$x^2 - y^2 + z^2 - 4x - 2y - 2z = 4$$

$$\begin{aligned} (x-2)^2 - (y+1)^2 + (z-1)^2 \\ = 4 - 4 + 1 - 1 \\ = 0. \end{aligned}$$

$$X = x-2 \quad Y = y+1 \quad Z = z-1$$

$$X^2 - Y^2 + Z^2 = 0$$



Vertex at
(0, 0, 0)

For x, y, z vertex at $(2, -1, 1)$

Ex. Sketch plane by

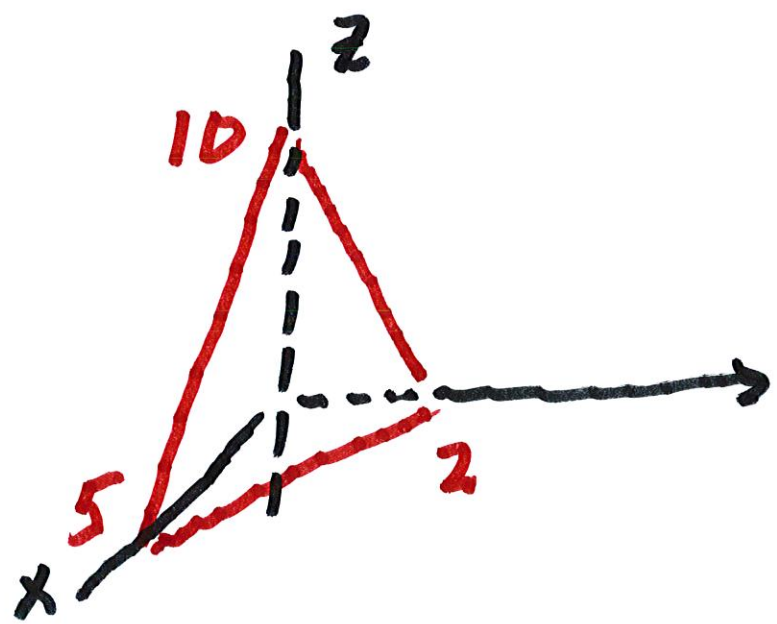
Using x , y , and z intercepts

$$2x + 5y + z - 10 = 0$$

$$x: \quad 2x + 0 + 0 - 10 = 0 \rightarrow x = 5$$

$$y: \quad 0 + 5y + 0 - 10 = 0 \rightarrow y = 2$$

$$z: \quad 0 + 0 + z - 10 = 0 \rightarrow z = 10$$



Ex. Find the saddle point

the surface

$$4x^2 - z^2 + 8x - 2z + 4y = 4$$

$$4(x+1)^2 - (z+1)^2 + 4y$$

$$= 4 + 4 - 1 = 7$$

$$\therefore 4y = (z+1)^2 - 4(x+1)^2 + 7$$

Center is at $(-1, \frac{7}{4}, -1)$

Ex.8 A force is given by

$$\text{a vector } \vec{F} = 3\vec{i} + 4\vec{j} + 5\vec{k}$$

and moves a particle from
the point $P(2, 1, 0)$ to

$Q(4, 6, 2)$. Find the work done.

$$\vec{PQ} = \langle 2, 5, 2 \rangle \quad W = \vec{F} \cdot \vec{D}$$

$$W = \langle 3, 4, 5 \rangle \cdot \langle 2, 5, 2 \rangle$$

$$= 6 + 20 + 10 = 36$$

Ex 9 Find the scalar projection

and vector projection of

$$\vec{b} = \langle 1, 4 \rangle \text{ onto } \vec{a} = \langle 3, 2 \rangle$$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$$

$$= \frac{11}{13} \langle 3, 2 \rangle$$

scalar eq'n is $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{11}{\sqrt{13}}$

Ex 10. Find the sine of the angle between $\vec{v} = \langle 2, -1, 3 \rangle$

and $\vec{w} = \langle 3, 2, 1 \rangle$.

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} = \sin \theta$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 2 & 1 \end{vmatrix} = \langle -7, 7, 8 \rangle$$

$$|\vec{a} \times \vec{b}| = \sqrt{49 + 49 + \frac{64}{49}} = \sqrt{162} = 9\sqrt{2}$$

$$7 \cdot \sqrt{3}$$

$$\equiv$$

$$|\vec{a}| = \sqrt{14}, \quad |\vec{b}| = \sqrt{14}$$

$$\therefore \sin \theta = \frac{9\sqrt{2}}{\sqrt{14}\sqrt{14}} = \frac{9\sqrt{2}}{14}$$

$$= \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

Ex. 6 Sketch

$$z - x^2 + y^2 + 4x + 6 = 0$$

$$z - (x-2)^2 + y^2 - 6 = -4$$

$$z - (x-2)^2 - y^2 = 2$$

This is a paraboloid



$$z = 2 + X^2 - Y^2$$

Curves down in y ,

Curves up in x .

$$\therefore \vec{r} = (0, 1, 0) + t(1, -2, 1)$$

$$x = t, \quad y = 1 - 2t, \quad z = t$$

Ex 2. Find the plane that
contains

$$P(1, 2, 3) \quad Q(2, -1, 4) \quad R(4, 1, 0)$$

$$\vec{PQ} = (1, -3, 1) \quad \vec{PR} = (3, -1, -3)$$

\vec{PQ} and \vec{PR} are both

tangent to plane.

$$\therefore \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 1 \\ 3 & -1 & -3 \end{vmatrix}$$

$$= 10\vec{i} + 6\vec{j} + 8\vec{k}$$

$$\text{or } 5\vec{i} + 3\vec{j} + 4\vec{k}$$

Note that $(1, 2, 3)$ is
in plane

$$5(x-1) + 3(y-2) + 4(z-4) = 0.$$