Math 341 Study Guide for Exam 2

1. You must memorize the proofs of these theorems from the textbook
   (a) Maximum-Minimum Theorem on p. 136
   (b) Mean Value Theorem on p. 173
2. You must be able to prove these theorems
   (a) Theorem 7.4.10 for continuous functions, on p. 231: Show that if a function f on the interval $I = [a, b]$ is continuous, then f is Darboux integrable on I. Your proof must use Uniform Continuity and the Integrability Criterion to show that if f is continuous then f is integrable. In your proof you should estimate $\sum_{k=1}^{n}(M_k - m_k)(x_k - x_{k-1})$.
   (b) Theorem 6.1.2 on p. 163: Show that if $f : I \to \mathbf{R}$ has a derivative at $c \in I$ then f is continuous at c.
   (c) Theorem 6.2.5 on p.174: Suppose that f is continuous on the closed interval $I = [a, b]$, that f is differentiable on the open interval (a, b), and that $f'(x) = 0$ for $x \in (a, b)$. Show that f is constant on I.
   (d) Show that if $g(x)$ is an anti-derivative of f, then $g(x) - g(a) = \int_a^x f(t)dt$. This is a corollary to Theorem 7.3.6 on p. 219.
3. You must be able to state these theorems from the textbook
   (a) Boundedness Theorem p.135
   (b) Uniform Continuity Theorem p. 143
   (c) Interior Extremum Theorem on p. 172
   (d) State the First Form of the Fundamental Theorem of Calculus when f is continuous on $[a, b]$. This means that you can let $E = \emptyset$ and omit part (c) from the statement of the theorem on p. 216.
   (e) State the Second Form of the Fundamental Theorem of Calculus, this is theorem 7.3.5 on p. 219
   (f) Taylor's Theorem with the Remainder p. 223

4. Review questions and problems

   (a) Define precisely $U(f,P), L(f,P), U(f)$ and $L(f)$. When is f Darboux integrable?

   (b) Evaluate $\lim_{x\to 0^+}(x)^{\frac{1}{x}}$ Hint: Apply ln to the function.

   (c) Evaluate $\lim_{x\to\infty}(1+\frac{2}{x})^{3x}$

   (d) For which rational numbers r does the function $f(x) = x^r sin(\frac{1}{x})$ satisfy $f'(0) = 0$.

   (e) Evaluate $\lim_{x\to\infty}\frac{\sqrt{x}-5}{\sqrt{x}+3}, x > 0$

   (f) If f and g are both differentiable on $\mathbf{R}$, what is $(g \circ f)'(c)$?

   (g) Section 5.3, page 140. 13

   (h) Section 5.4, page 148. 9

   (i) Section 6.1, page 171. 2

   (j) Section 6.4, page 196. 9