

Review Problems for Math 341

1a. Given a set S of real numbers, define $\sup S = u$

b. Show that for any $\epsilon > 0$,

there is a number $x_\epsilon \in S$

such that $u - \epsilon < x_\epsilon \leq u$

page 37-38.

2. Define the Nested

Interval Property. p. 48

3. Suppose that (x_n) converges to x and (y_n) converges to y . Show that $(x_n y_n)$ converges to xy .

p. 61, 62.

4. Show that if (x_n) is an increasing sequence and that $x_n \leq M$ for all n , then there is a number $L \leq M$ such that $\lim_{n \rightarrow \infty} x_n = L$

p. 71, 72

5. Suppose $\sum_{n=1}^{\infty} x_n$ is a series

with $x_n \geq 0$ such that

$$\sum_{n=1}^N x_n \leq M \text{ for all } N=1, 2, \dots$$

Show there is an $L \leq M$

such that $\sum_{n=1}^{\infty} x_n$ converges

to L . p. 98

6. Use Newton's Method

one time with an initial

guess of 2 to find the

approximate value of $\sqrt{3}$

p. 194

7. Define the Thomae function

by setting $f(x) = \frac{1}{q}$, when

$x = p/q$ in lowest terms,

and by $f(x) = 0$, when x

is irrational. Show that

f is continuous at x if

x is irrational and f is

discontinuous at x if

x is rational. p. 127, 128

8. Suppose that f is an increasing bounded function on (a, b) . Show there is an L such that $\lim_{x \rightarrow b^-} f(x) = L$.

p. 117, 118

9. Show that $f(x) = \sqrt{x}$ is Lipschitz on the interval $[a, \infty)$, where $a > 0$.

p. 143, 144

10. Show that if f is Lipschitz on any interval, then f is uniformly continuous.

p. 143

11. Find all functions that satisfy $|f(x) - f(y)| \leq |x - y|^2$

p. 162

12. Suppose that f is differentiable at x_0 and that $f'(x_0) \neq 0$

Calculate $\left(\frac{1}{f}\right)'$ at x_0 .

p. 115, 164

13. Let n be a positive integer and let $b > 0$. Use L'Hopital's

Rule to show that

$$\lim_{x \rightarrow \infty} \left(\frac{x^n}{b^x} \right) = 0, \text{ i.e.}$$

$|x^n| \leq b^x$, if x is sufficiently large.

p. 182

14. Use $\ln x$ and L'Hopital's Rule

to show that $\lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^{bn}$

Set $x = \frac{1}{n}$.

$$= e^{ab}$$

p. 183

15. If the Taylor polynomial of order 3 of $\ln(1+x)$ is used to calculate the approximate value of $\ln \frac{3}{2}$, what is the maximum error allowed by the Remainder Estimate

16. Show that on every interval $[-d, d]$, the Taylor series of $\cos x$ converges to $\cos x$.

17. Define $g(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & \text{if } x=0 \end{cases}$.

Is g continuous at 0? Is

g differentiable at 0? Is

$g \in R[0, 1]$. Is g uniformly
continuous on $[0, 1]$?

p. 130, 162, 212

18. Suppose f is differentiable
at all $x \in (a, b)$, If $f'(x) > 0$
in (a, b) , show f is increasing.

p. 174

19. When f is a bounded function on $[a, b]$ and when P is a partition of $[a, b]$ that is tagged, what is $S(f; P)$?

p. 200

20. What is the definition of the statement $f \in R[a, b]$?

p. 201

21. State the Integrability

Criterion for a function to
be Darboux - integrable.

22. Let $g(x) = \begin{cases} 3 & \text{if } 0 \leq x < 2 \\ 1 & \text{if } 2 \leq x \leq 4 \end{cases}$
p. 229

Use the criterion above with
a partition P having just 4
points to show g is Darboux
integrable. p. 229, p. 228

23. Use the above criterion
to show that any continuous
function f on $[a, b]$ is Darboux
integrable. p. 229, 228

24. To solve the differential
equation $y'(x) = f(x, y(x))$
with $y(x_0) = y_0$, we defined a
sequence of curves $y_n(x)$.

How are curves $y_n(x)$

defined?

(class notes)
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