12.2 Components

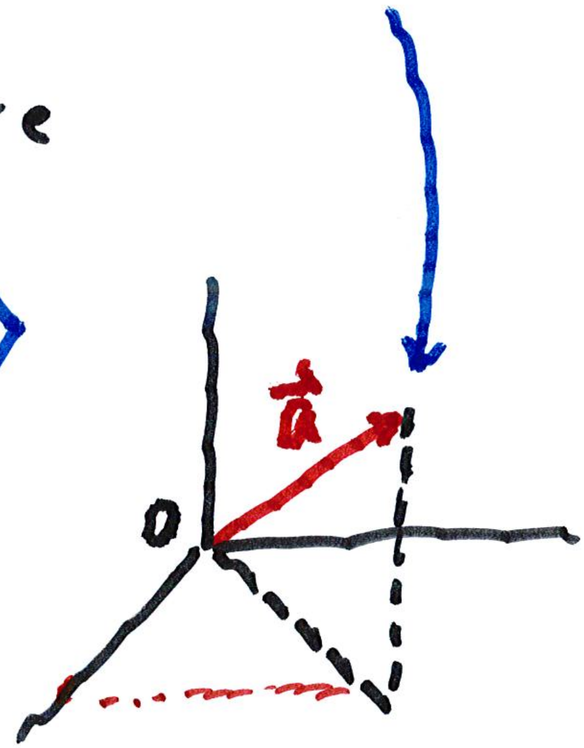
If $\vec{a}$ is a vector in $\mathbb{R}^3$,

and if $O = (0,0,0)$ is the

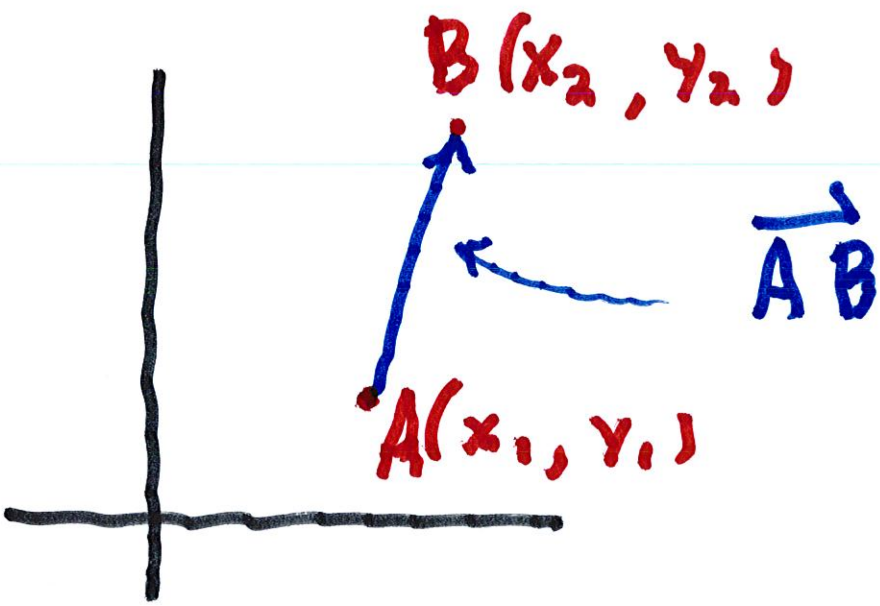
initial pt. and $(a_1, a_2, a_3) =$
$\vec{a}$ = term. pt.,

then we can write

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$



In $\mathbb{R}^2$,



$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

Displacement Vector

The length of $\vec{a} = \langle a_1, a_2, a_3 \rangle$

is $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

If $\vec{a} = \langle a_1, a_2, a_3 \rangle$

and $\vec{b} = \langle b_1, b_2, b_3 \rangle$,

then $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

and $\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

Also $c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$

It's similar in $\mathbb{R}^2$

Ex. If $\vec{a} = \langle 2, 3, 1 \rangle$ and

$\vec{b} = \langle 3, 1, -2 \rangle$, find

$2\vec{a} - 3\vec{b}$ $\qquad 2\vec{a} = \langle 4, 6, 2 \rangle$

$3\vec{b} = \langle 9, 3, -6 \rangle$

$\Rightarrow 2\vec{a} - 3\vec{b} = \langle -5, 3, 8 \rangle$

Properties of Vectors:

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

3. $\vec{a} + \vec{0} = \vec{a}$ $\qquad \vec{0} = \langle 0, 0, 0 \rangle$

4. $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$

5. $\vec{a} + (-\vec{a}) = \vec{0}$

6. $(c+d)\vec{a} = c\vec{a} + d\vec{a}$

7. $(cd)\vec{a} = c(d\vec{a})$

8. $1\vec{a} = \vec{a}$

Standard Basis Vectors

We set $\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$ $\vec{k} = \langle 0, 0, 1 \rangle$

$\therefore \vec{a} = \langle a_1, a_2, a_3 \rangle$

$= \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0, 0, a_3 \rangle$

$= a_1 \langle 1, 0, 0 \rangle + a_2 \langle 0, 1, 0 \rangle + a_3 \langle 0, 0, 1 \rangle$

$= a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$

Ex. If $\vec{a} = \vec{i} + 2\vec{j}$

and $\vec{b} = -3\vec{i} + 3\vec{j}$,

then $2\vec{a} + 3\vec{b}$

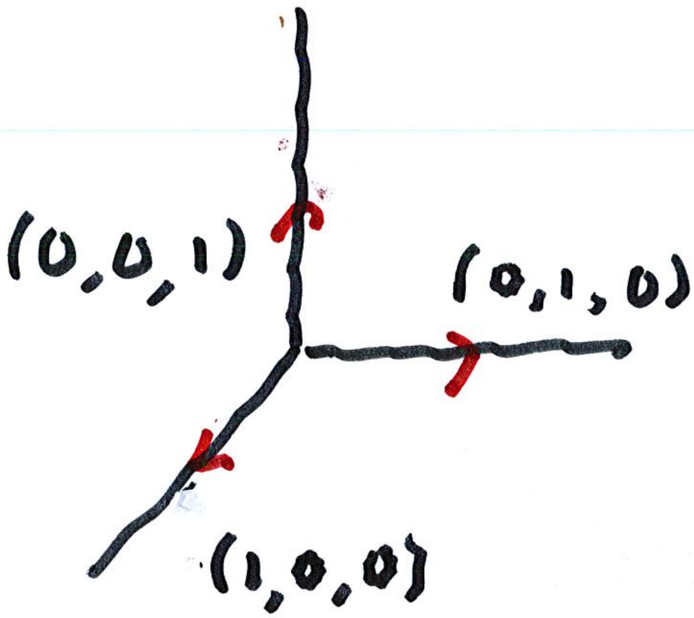
$= 2(\vec{i} + 2\vec{j}) + 3(-3\vec{i} + 3\vec{j})$

$= -7\vec{i} + 13\vec{j}$

~~Unit vectors: If $\vec{a} \neq \vec{0}$.~~

A unit vector is a vector of length = 1

$\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors



If $\vec{a} \neq \vec{0}$, then the vector $\dfrac{\vec{a}}{|\vec{a}|}$ is a unit vector that points in the same direction as $\vec{a}$

In fact:

$$\left|\frac{\vec{a}}{|\vec{a}|}\right| = \left|\frac{1}{|\vec{a}|}\vec{a}\right|$$

$$= \left|\frac{1}{|\vec{a}|}\right| |\vec{a}|$$

$$= \frac{1}{|\vec{a}|}|\vec{a}| = 1$$

$\therefore \frac{\vec{a}}{|\vec{a}|}$ is a <u>unit vector</u>.

Ex. Find a unit vector $\vec{u}$ that points in the same dir. as $\vec{v} = \vec{i} - 2\vec{j} + 2\vec{k}$

$$|\vec{v}| = \sqrt{1^2 + (-2)^2 + 2^2}$$

$$= \sqrt{1+4+4} = 3.$$

$$\therefore \vec{u} = \frac{1}{3}\left(\vec{i} - 2\vec{j} + 2\vec{k}\right)$$

$$= \frac{1}{3}\vec{i} - \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$$

Ex. Find a vector of length 3 that points in the opposite direction of $\vec{v} = 3\vec{i} + \vec{j}$

Set $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{3\vec{i} + \vec{j}}{\sqrt{9+1}}$

$= \dfrac{3}{\sqrt{10}}\vec{i} + \dfrac{1}{\sqrt{10}}\vec{j}$

Note that $-\vec{u} = -\dfrac{3}{\sqrt{10}}\vec{i} - \dfrac{1}{\sqrt{10}}\vec{j}$

points in the opp. direction

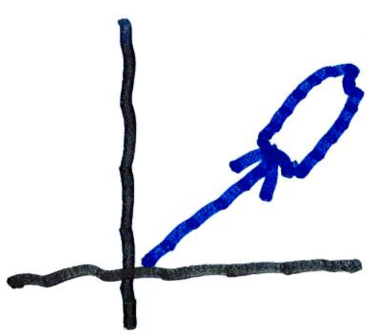
Now set

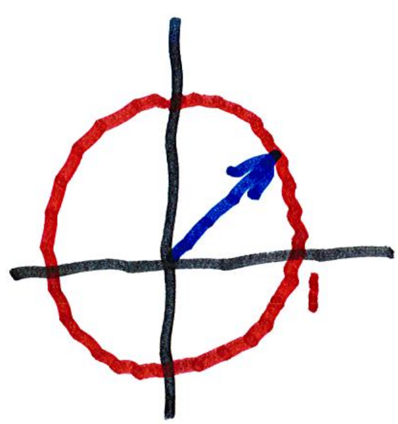
$$\vec{w} = 3\left(-\frac{3}{\sqrt{10}}\vec{i} - \frac{1}{\sqrt{10}}\vec{j}\right)$$

$$= -\frac{9}{\sqrt{10}}\vec{i} - \frac{3}{\sqrt{10}}\vec{j}$$

Ex. A man is walking [redacted] N [redacted] on the deck of a ship at 2 mph. The ship is moving NE at 10 mph.

Find the velocity vector of the man relative to the surface of the water.





~~$\vec{V}_{ship} = 10$~~

$$\vec{V}_{ship} = 10\left(\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{i}\right)$$

$$= 5\sqrt{2}\,\vec{i} + 5\sqrt{2}\,\vec{j}$$

$$\vec{V}_{man} = 0\vec{i} + 2\vec{j}$$

(rel. to ship)

$$\vec{V} = \vec{V}_{ship} + \vec{V}_{man}$$

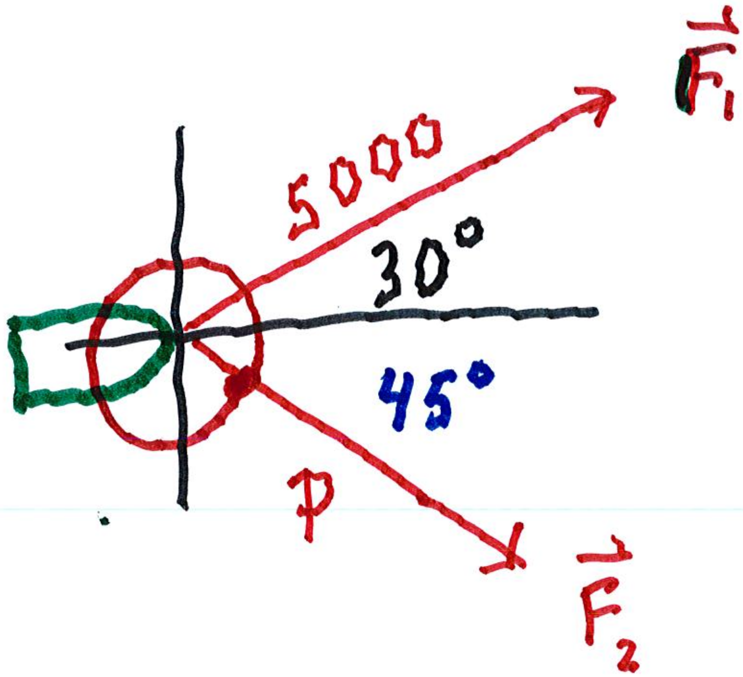
$$= 5\sqrt{2}\,\vec{i} + \left(5\sqrt{2} + 2\right)\vec{j}$$

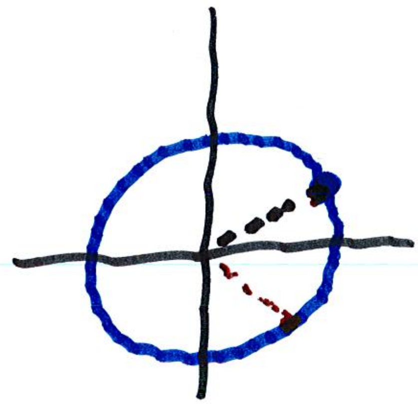
Ex. One tug pulls a ship in direction 30° north of east, with a force of 5000 lb.

~~Ano~~

Another pulls ship in SE direction with force of P lbs.

What should P be for ship to go in E direction?





$$\vec{F}_1 = 5000\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$$

$$\vec{F}_2 = P\left(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right)$$

Combined force is

$$\left(2500\sqrt{3} + \frac{P}{\sqrt{2}}\right)\vec{i} + \left(2500 - \frac{P}{\sqrt{2}}\right)\vec{j}$$

We need coefficient of

$\vec{j}$ to $= 0$.

$\therefore \quad 2500 - \frac{P}{\sqrt{2}} = 0$

or $\quad P = \underline{\underline{2500\sqrt{2}}}$

Recall $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

or $\quad \cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$