Polar Coord. part 2.

Express in polar coordinates:

1. $y = 3 + 2x$

$$r\sin\theta = 3 + 2r\cos\theta$$

$$r(\sin\theta - 2\cos\theta) = 3$$

$$r = \frac{3}{\sin\theta - 2\cos\theta}$$

2. $x^2 + y^2 = 4y$

$$x^2 + y^2 - 4y = 0$$

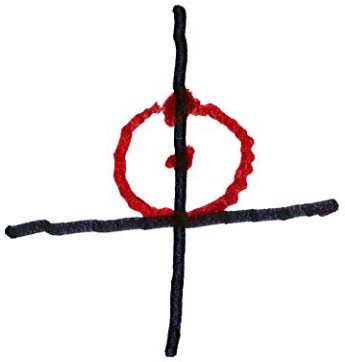
Complete the square:

$$x^2 + (y-2)^2 = 4$$

A circle of radius 2 with

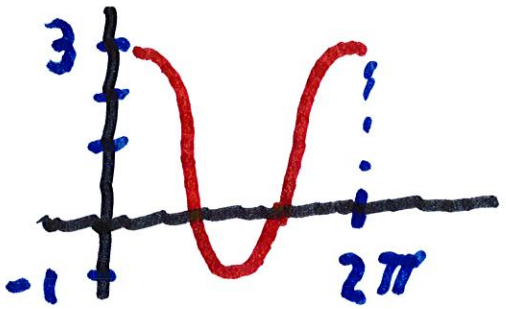
center = $(0, 2)$

$r = 4\sin\theta$



3. Sketch the curve:

$$r = 1 + 2\cos\theta$$



Find θ where

$r(\theta) = 0$

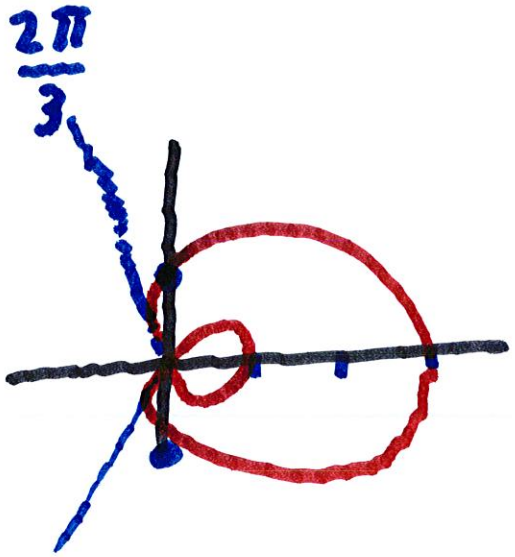
$\rightarrow \quad 1 + 2\cos\theta = 0$

$2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$

$\therefore \quad \theta = \frac{2\pi}{3} \quad$ or $\quad \frac{4\pi}{3}$

If $\theta = 0 \rightarrow r = 3$ $\quad \theta = 2\pi \rightarrow r = 3$



$\frac{2\pi}{3} < \theta < \frac{4\pi}{3}$

$\rightarrow r(\theta) < 0$

4. $r = 2 + \sin\theta$

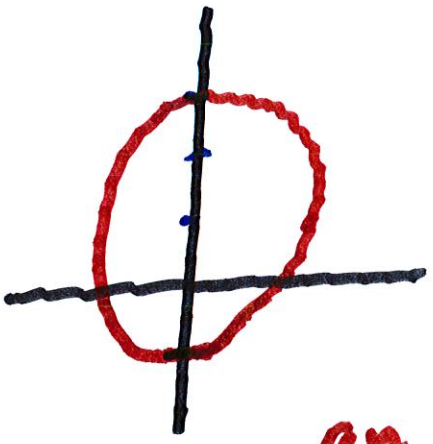
$r(\theta) \geq 1$

$+1 \leq r \leq 3$

$\frac{r}{2} =$

$r = 2\left(\frac{1 + \sin\theta}{2}\right)$

$r = 2\left(1 + \frac{\sin\theta}{2}\right)$

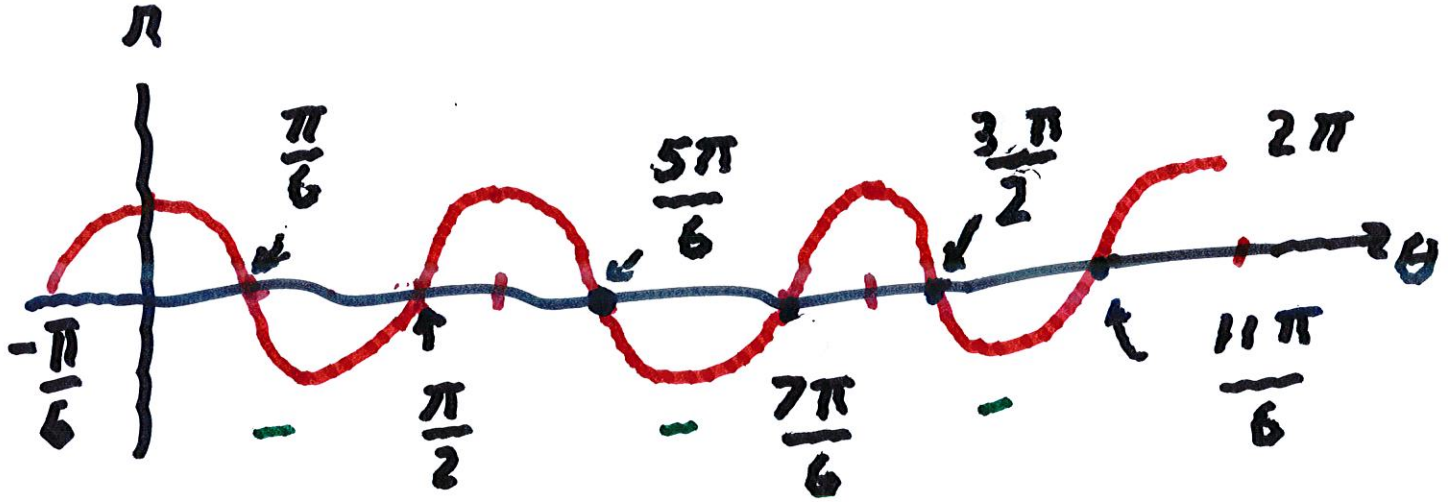


an oval

5. Sketch the curve

$$r = 2\cos 3\theta$$

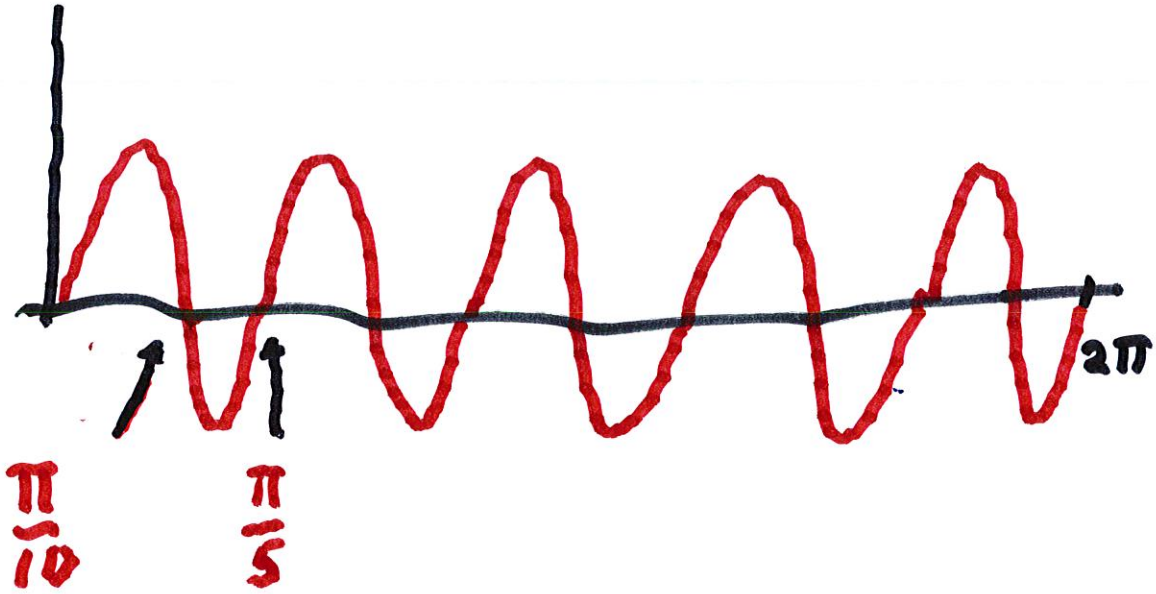
First sketch the curve $r = 2\cos 3\theta$ in Cartesian coordinates

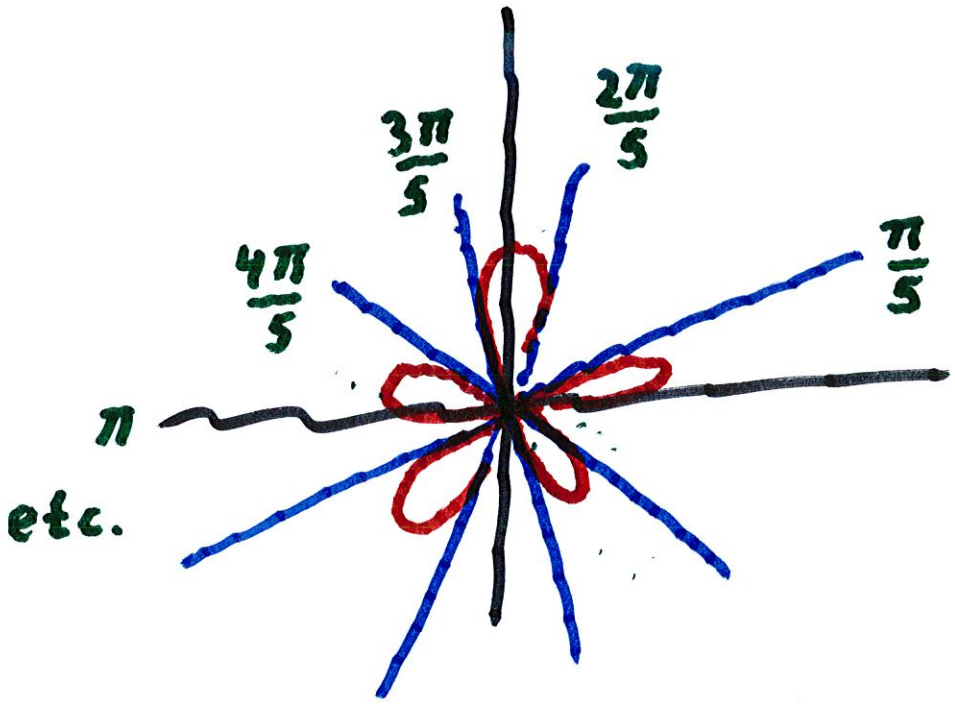


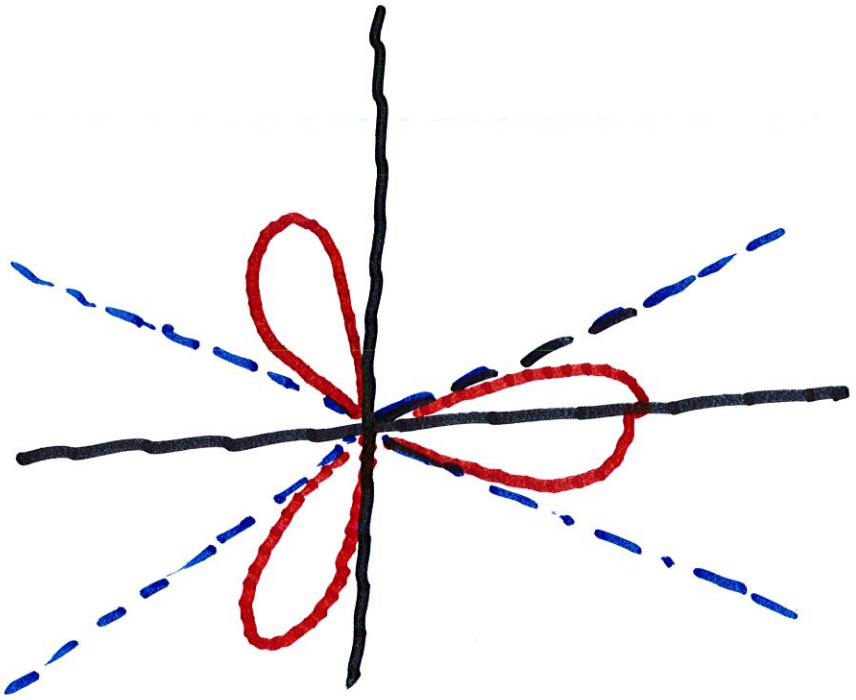
$\cos\frac{\pi}{2} = 0$ Need $3\theta = \frac{\pi}{2}$

or $\theta = \frac{\pi}{6}$

6. Sketch $r = \sin 5\theta$

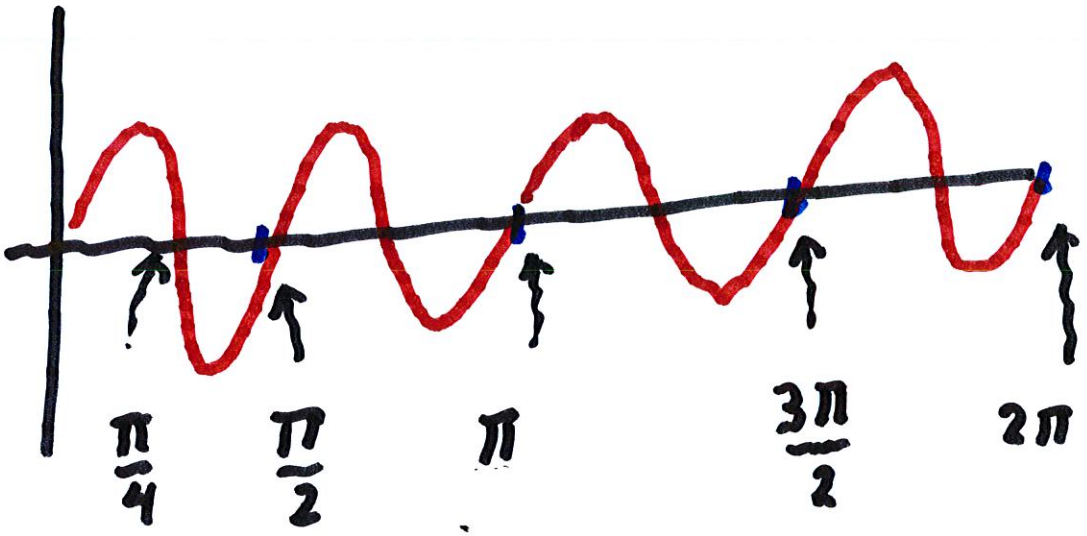


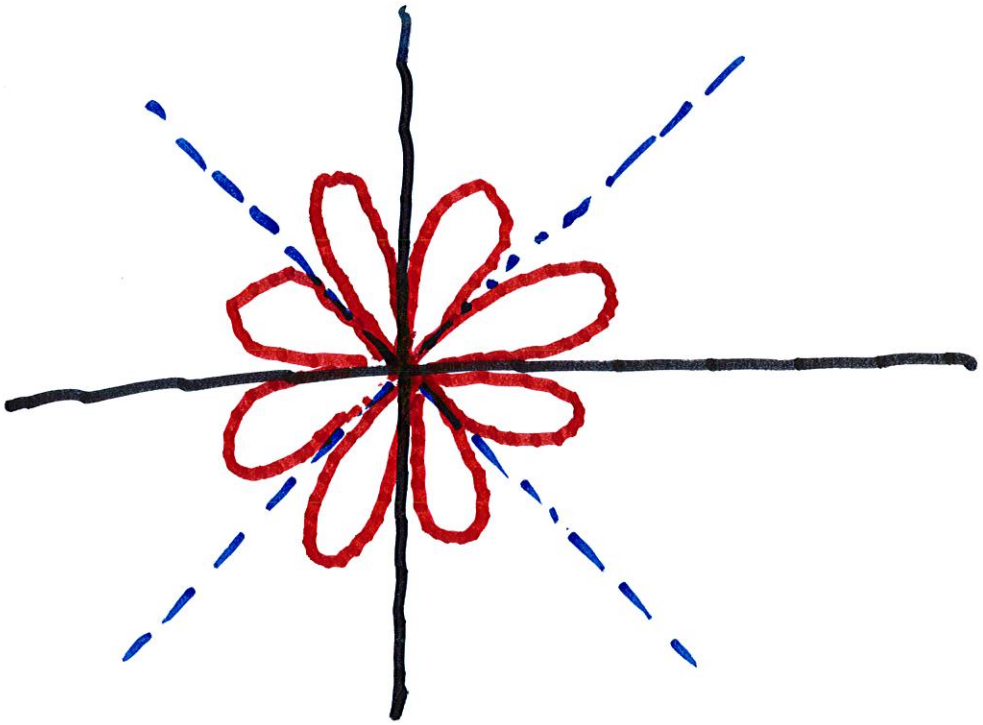




As θ goes from 0 to 2π, each point in each loop occurs twice.

6 Sketch $r = 2\sin 4\theta$

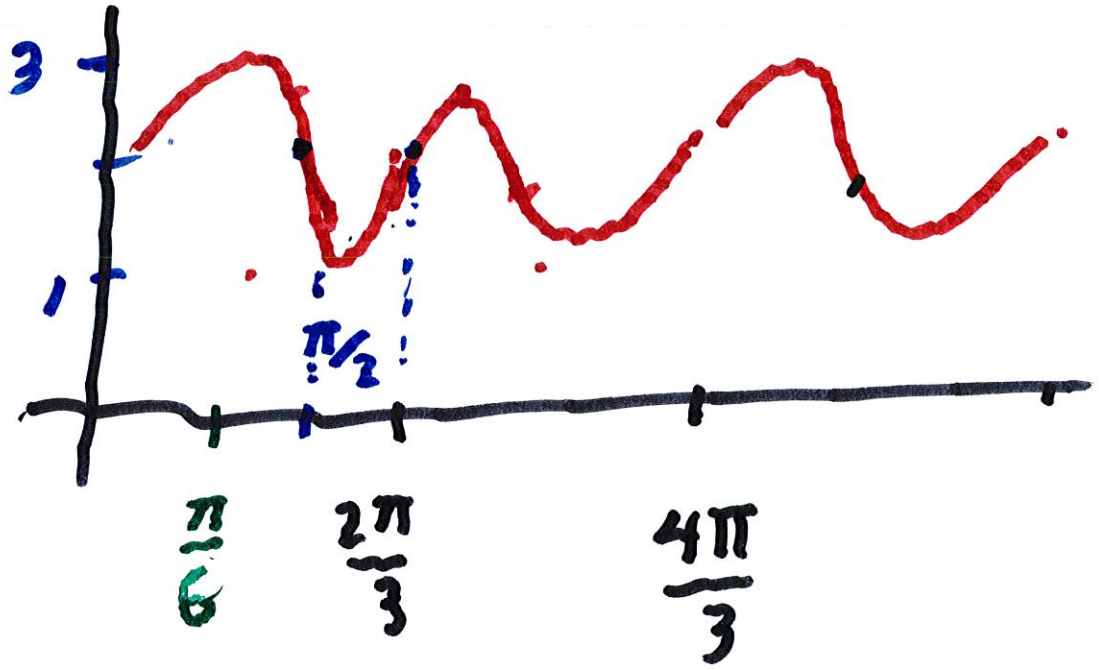


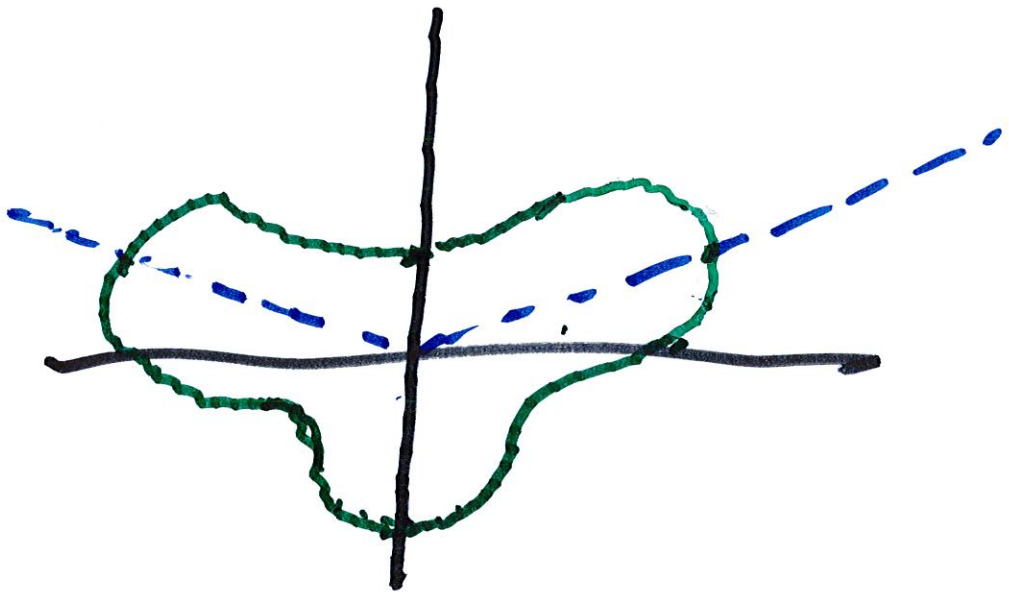


7 Sketch $r = 2 + \sin 3\theta$

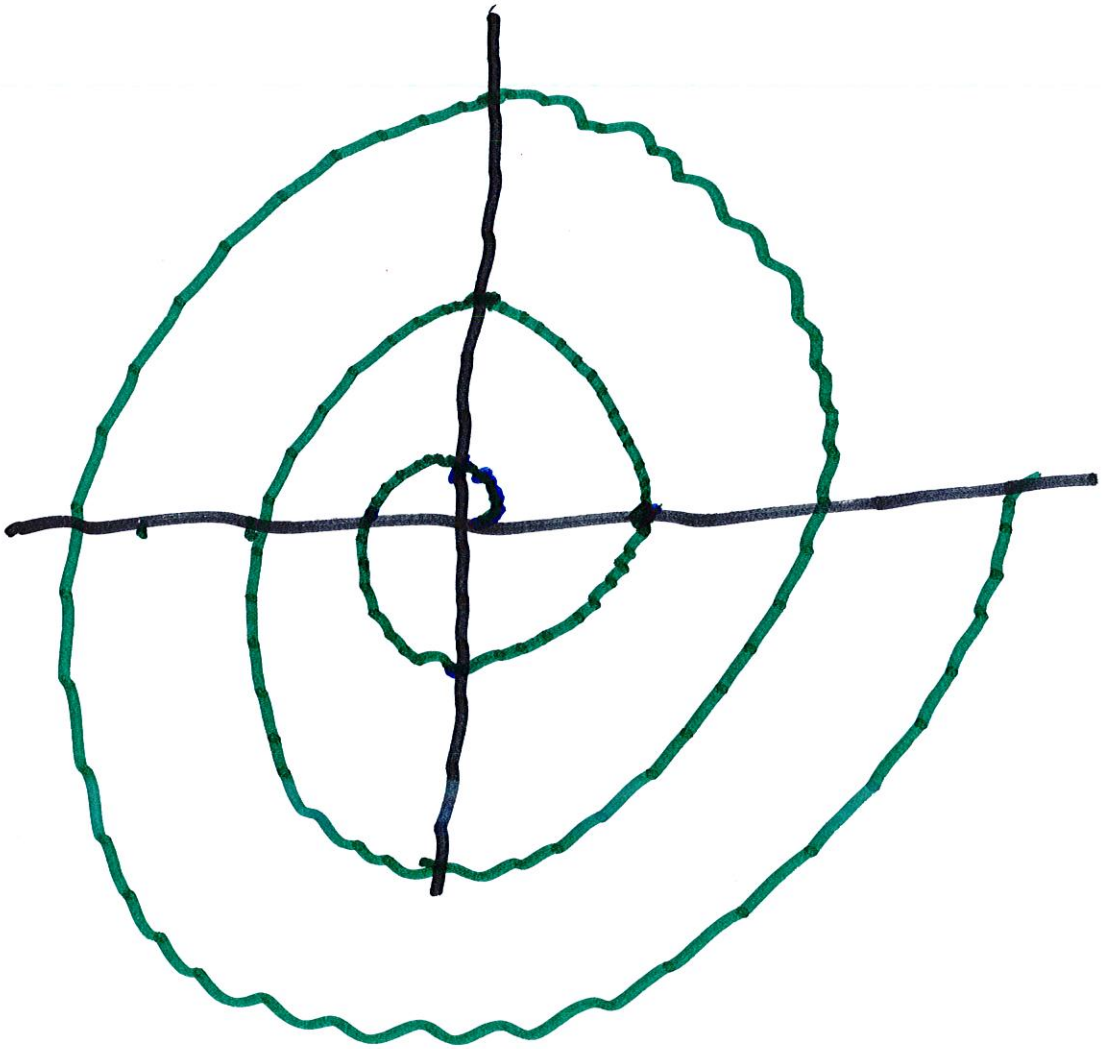
Note $r > 0$ max = 3

min = 1





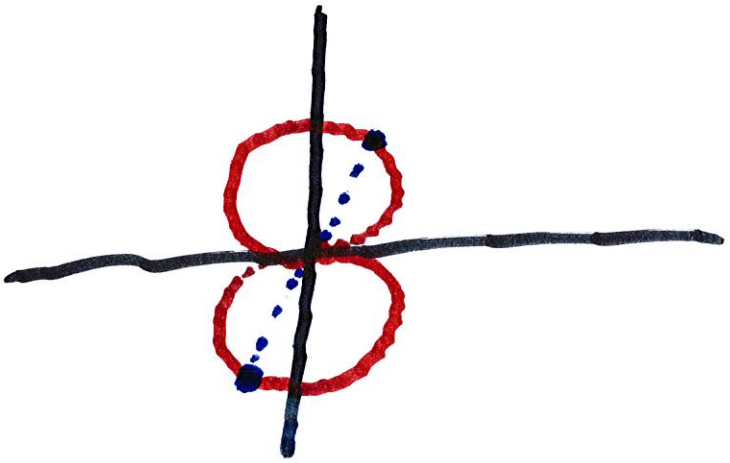
8. Sketch $r = \theta$ for $0 \leq \theta \leq 6\pi$



9. Sketch $r^2 = \sin\theta$

Note this has no solution when $\sin\theta < 0$

$r = \pm\sqrt{\sin\theta}$ when $\sin\theta \geq 0$



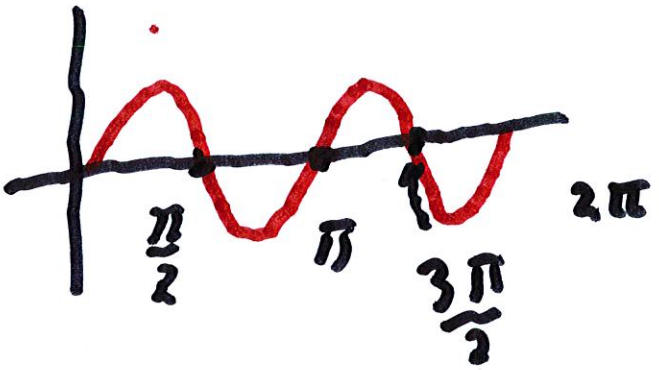
10. Sketch $r^2 = \sin 2\theta$

$r = \pm\sqrt{\sin 2\theta}$ when $\sin 2\theta \geq 0$

10. Sketch $r^2 = \sin 2\theta$

Note $\sin 2\theta > 0$ if $0 < \theta < \frac{\pi}{2}$

or if $\pi < \theta < \frac{3\pi}{2}$



$\therefore r = \pm\sqrt{\sin 2\theta}$