Ex. 5 Find the shortest distance from $(1,0,-2)$ to the plane

$$x + 2y + z = 4.$$

$$d = \sqrt{(x-1)^2 + y^2 + (z+2)^2}.$$

We can minimize the square by minimizing the square

$$d^2 = f(x,y) = (x-1)^2 + y^2 + (\underbrace{4 - x - 2y}_{z} + 2)^2$$

$$f(x,y) = (x-1)^2 + y^2 + (6-x-2y)^2$$

$$f_x = 2(x-1) + 2(6-x-2y)(-1)$$

$$=$$

$$f_x = 4x + 4y - 14 = 0.$$

$$f_y = 2y + 2(6-x-2y)(-2)$$

$$= 10y + 4x - 24 = 0$$

$$\therefore \quad 4x + 4y = 14$$

$$4x + 10y = 24$$

$$\rightarrow \quad 6y = 10 \rightarrow y = \cancel{\tfrac{10}{3}} \; \frac{10}{6} = \frac{5}{3}$$

$$\therefore \quad 4x = 14 - 4y = 14 - \frac{20}{3} = \frac{22}{3}$$

$$\rightarrow x = -\frac{22}{12} = -\frac{11}{6}$$

$\therefore$ Only crit. point is $\left(\frac{11}{6}, \frac{5}{3}\right)$

Intuitively, this is a local minimum.

To find an absolute minimum

on a ~~K~~ closed bounded set:

1. Find the values at the critical points.

2. Find the extreme values of f on the boundary of D.

3. Compare the values from (1) and (2)

33 Find the absolute maximum and minimum points of

$$f(x,y) = x^4 + y^4 - 4xy + 2$$

$$D = \{ (x,y) \mid 0 \leq x \leq 3,\ 0 \leq y \leq 2 \}$$

$$f_x = 4x^3 - 4y = 0 \rightarrow y = x^3$$

$$f_y = 4y^3 - 4x \qquad x = y^3$$

$$x = y^3 = (x^3)^3 \rightarrow x = 0 \text{ or } 1 = x^8$$

$$x = x^9 \rightarrow 1 = x^8$$

If $x=0 \to y=0$

If $x=-1$

$\downarrow$

$y=(-1)^8=1$

If $x=1$

$\downarrow$

$y=1$

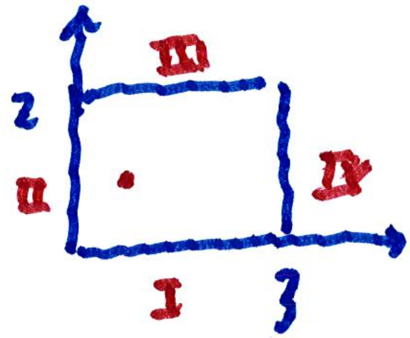
$\therefore$ 3 crit. points

$(0,0)$ $(1,1)$ $(-1,1)$

$\uparrow$ not in interior

$\downarrow$ in int. of D

$f(1,1)=2-4+2=0$



$f(x,0) = x^4 + 0 + 0 + 2 = x^4 + 2$

I $\quad \underline{f(3,0) = 83} \qquad f(0,0) = 2$

II $\quad f(0,y) = y^4 + 2 \leq 18 \qquad \underline{2^4 + 2 = 18}$

$y=0 \to 2$

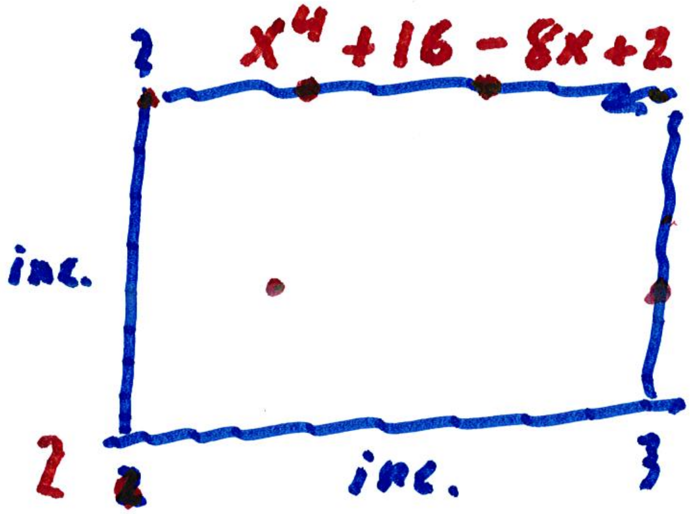
III $\quad f(x,2) = x^4 + 16 - 8x + 2 \approx 87$

$-4xy$

Find max of $x^4 + 16 - 8x + 2$

$y'(x) = 4x^3 - 8$

$\rightarrow x^3 = 2 \rightarrow x = \sqrt[3]{2}$



$81 + y^4 - 12y + 2$

$\boxed{IV}$ $4y^3 - 12 = 0$

$y^3 = 3$

It appears that $f(1,1)$ is the minimum value:

$f(1,1) = 0$

#31 $f(x,y) = x^2 + y^2 - x^2y + 4$

$D = |x| \leq 1, \ |y| \leq 1$

$f_x = 2x - 2xy \qquad f_y = 2y - x^2 = 0$

$= 2x(1-y) = 0 \qquad \rightarrow y = \dfrac{x^2}{2}$

$x = \sqrt{2}$

$x = 0 \rightarrow y = 0 \quad (0,0) \quad (\sqrt{2}, \qquad x = \pm\sqrt{2}$

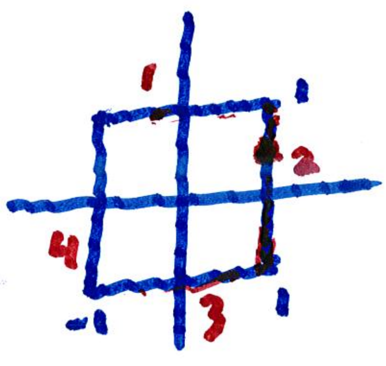
$(-\sqrt{2}, 1)$ $(\sqrt{2}, 1)$

For $(0,0)$

$$D = \begin{vmatrix} 2 & -2x \\ -2x & 2 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

$D = 4 - 0 \Rightarrow$ local min or a local max

$(0,0)$



Look at $(x,1)$ $-1 \leq x \leq 1$

1. $f(x,1) = x^2 + 1 - x^2 + 4 = 5$

$$f(1,y) = 1 + y^2 - y + 4$$
$$= y^2 - y + 3$$