Ex. 1 Sketch the surface

$$x^2 + 4y^2 = 4$$

horizontal trace $(z = c)$

is an ellipse, independent

of z.

Ex. 2 Sketch the surface

$$z = -y^2$$

ind. of x

a parabolic cylinder

Ex. 3

Sketch $x^2 + y^2 - 4z^2 = 4$

For each z, the horizontal trace is a circle (with $z = c$) of radius $\sqrt{4+4z^2}$

Each vertical trace is a hyperbola if $y = c$ (or $x = c$).

$$x^2 - 4z^2 = 4 - c^2$$

(hyperboloid of one sheet)

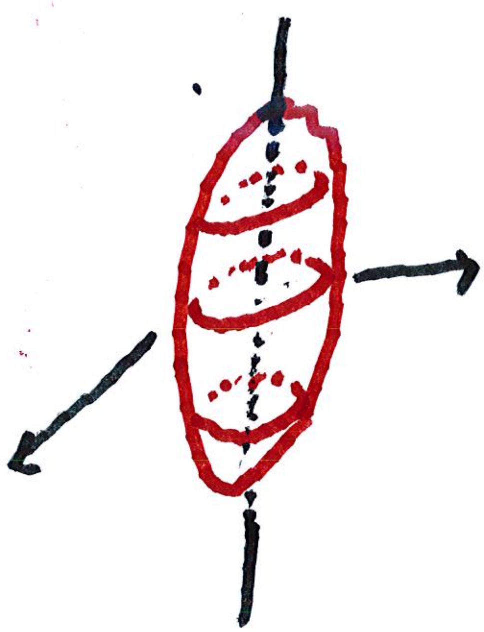
Ex. 4. Sketch the surface

$$x^2 + 4z^2 + y^2 = 4$$

Look at the horizontal

trace $x^2 + y^2 = 4 - 4z^2$

This is a circle if $z^2 < 1$



This is an ellipsoid.

Ex. 5 Sketch

$$x^2 - y^2 + 4z^2 = -16$$

If we look at the y-trace, we get $x^2 + 4z^2 = y^2 - 16$.

If $y^2 < 16$, then there is NO SOLUTION. But if $y^2 > 16$, the trace is an ellipse

This is a hyperboloid of 2 sheets

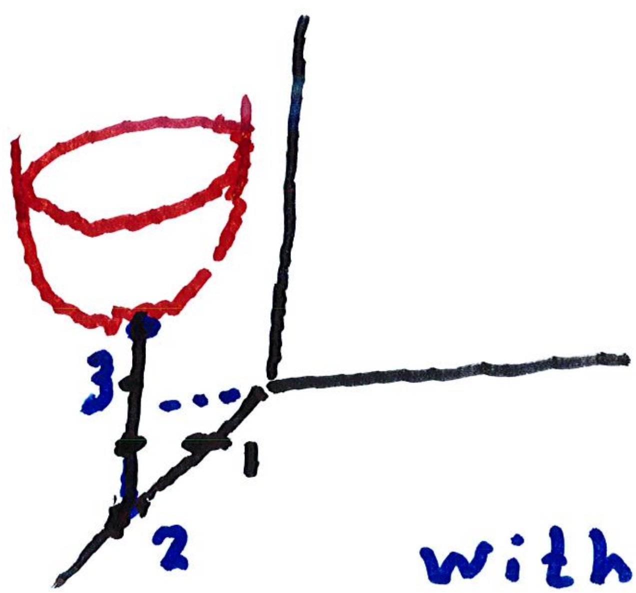
Ex. 6. Sketch

$$z - x^2 - y^2 + 4x - 7 = 0$$

$$z = x^2 - 4x + y^2 + 7$$

$$z = (x-2)^2 + y^2 + 7 + 4$$

$$z = (x-2)^2 + y^2 + 3$$

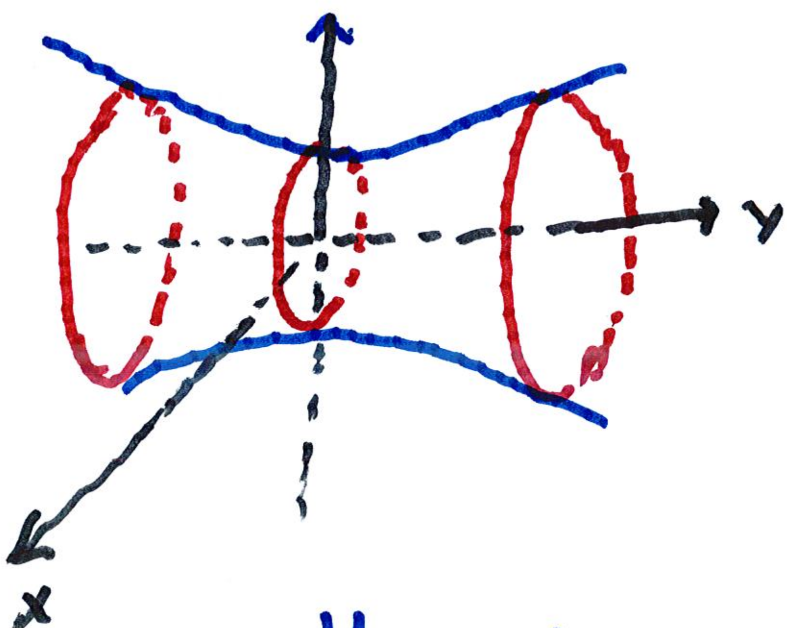


This is a paraboloid with vertex at $(2, 0, 3)$

Look at

$$X^2 - Y^2 + Z^2 = 2$$

$$X^2 + Z^2 = 2 + Y^2 > 0$$



Hyperboloid of 1 sheet

Ex.7 Find the equation for the points that are equidistant from $(-1, 0, 0)$ and the plane $x = 1$

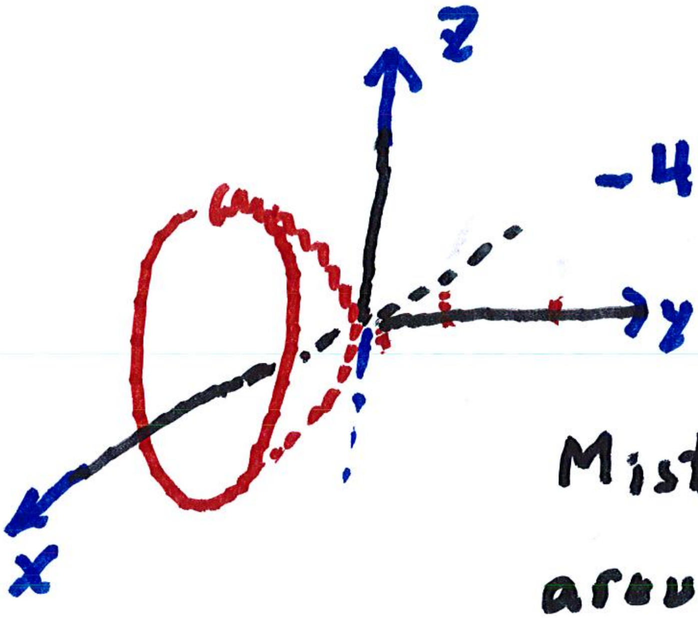
d_1 d_2

$$\sqrt{(x+1)^2 + y^2 + z^2} = 1 - x$$

$$(x+1)^2 + y^2 + z^2 = 1 - 2x + x^2$$

$$\rightarrow \quad y^2 + z^2 = -4x$$

A paraboloid that spins around the negative x-axis



$-4x = y^2 + z^2$

Mistake: Should go around neg. - x - axis

Ex8 Sketch

$x^2 - y^2 + z^2 - 4x - 2y - 2z = -8$

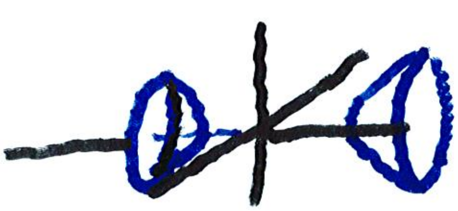
$(x-2)^2 - (y+1)^2 + (z-1)^2$

$= -8 + 4 - 1 + 1$

center = (2, -1, 1)

$= -4$

$X^2 + Z^2 = -4 + Y^2$ If $|y| > 2$,

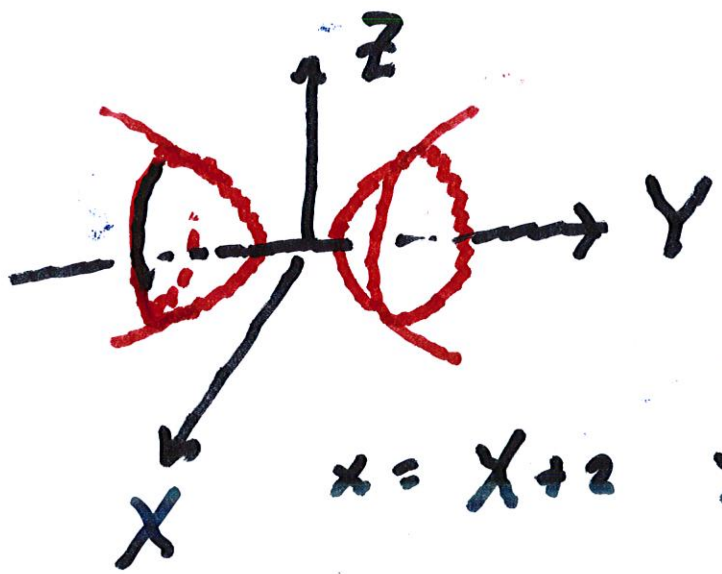


y-trace is a circle.

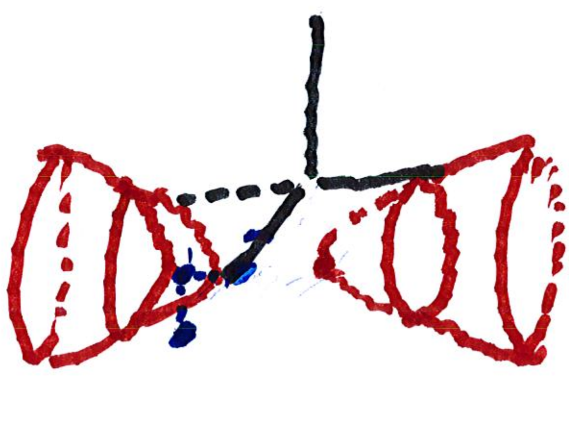
$\therefore \quad (x-2)^2 + (z-1)^2 = (y+1)^2 - 4$

Set $X = x-2$, $Y = y+1$, $Z = z-1$

$\rightarrow \quad X^2 + Z^2 = Y^2 - 4$



$x = X+2 \quad y = Y-1 \quad z = Z-1$



center at

$(2, -1, -1)$

Location of center NOT RIGHT!

Ex. 1. Find parametric equations of line that lies in intersection of

$x + y + z = 1$ $\quad$ $x + 2y + 3z = 2$

P_1 $\quad$ P_2

$\vec{n}_1 = \langle 1, 1, 1 \rangle$ $\quad$ $\vec{n}_2 = \langle 1, 2, 3 \rangle$

If $\vec{v}$ = direction of line L, then

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \vec{i} - 2\vec{j} + \vec{k}$$

Must find a point in L

Set $z = 0$

$$\left.\begin{aligned} x + y &= 1 \\ x + 2y &= 2 \end{aligned}\right\} \quad y = 1, \quad x = 0$$

$\rightarrow (0, 1, 0)$ is in L.

$$\therefore \langle x, y, z \rangle = (0, 1, 0) + t \langle 1, -2, 1 \rangle$$

$\rightarrow \quad x = t, \quad y = 1 - 2t, \quad z = t$

Ex. Sketch

$$x^2 - y^2 + z^2 - 4x + 2y + 8z = -19$$

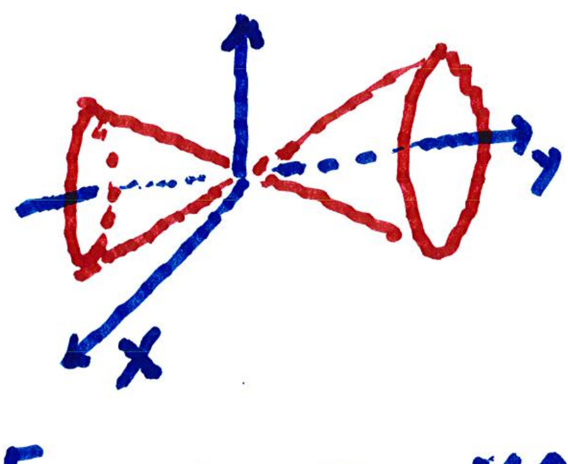
$$(x-2)^2 - (y-1)^2 + (z+4)^2$$

$$= -19 + 4 - 1 + 16$$

$$= 0.$$

$X = x-2 \quad Y = y-1 \quad Z = z+4$

$$X^2 - Y^2 + Z^2 = 0 \quad \text{(Cone)}$$



Vertex at $(0, 0, 0)$

For x, y, z vertex at $(2, 1, -4)$

Ex. Sketch plane by

using x, y, and z intercepts

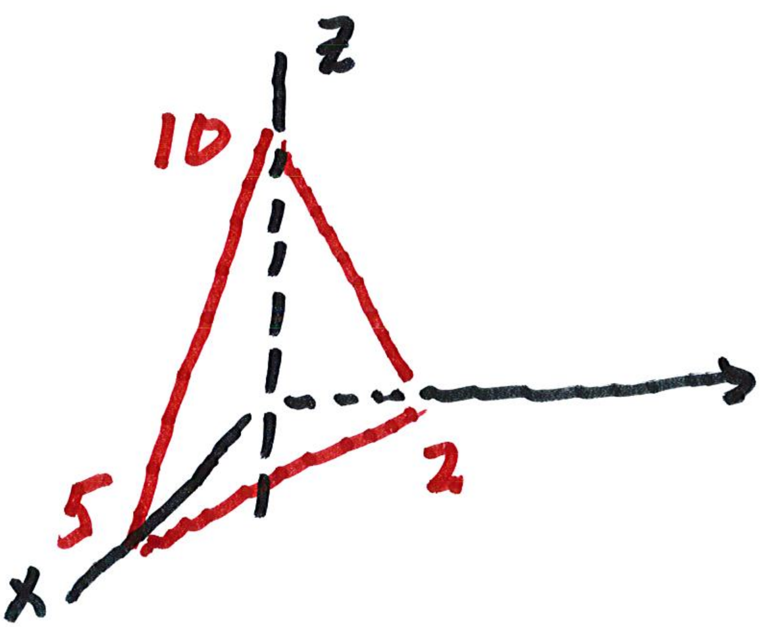
$$2x + 5y + z - 10 = 0$$

x: $2x + 0 + 0 - 10 \rightarrow x = 5$

y: $0 + 5y + 0 - 10 = 0$

$\rightarrow y = 2$

z: $0 + 0 + z - 10 = 0 \rightarrow z = 10$



Ex. Find the saddle point of the surface

$$4x^2 - z^2 + 8x - 2z + 4y = 4$$

$$4(x+1)^2 - (z+1)^2 + 4y$$
$$= 4 + 4 - 1 = 7$$

$$\therefore \quad 4y = (z+1)^2 - 4(x+1)^2 + 7$$

Center is at $\left(-1, \frac{7}{4}, -1\right)$

Ex.8 A force is given by a vector $\vec{F} = 3\vec{i} + 4\vec{j} + 5\vec{k}$ and moves a particle from the point $P(2, 1, 0)$ to $Q(4, 6, 2)$. Find the work done.

$\overrightarrow{PQ} = \langle 2, 5, 2 \rangle \qquad W = \vec{F} \cdot \vec{D}$

$W = \langle 3, 4, 5 \rangle \cdot \langle 2, 5, 2 \rangle$

$= 6 + 20 + 10 = 36$

Ex 9 Find the scalar projection and vector projection of

$\vec{b} = \langle 1, 4 \rangle$ onto $\vec{a} = \langle 3, 2 \rangle$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|a|^2} \vec{a}$$

$$= \frac{11}{13} \langle 3, 2 \rangle$$

---

scalar eq'n is $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \dfrac{11}{\sqrt{13}}$

Ex 10. Find the sine of the angle between $\vec{a} = \langle 2, -1, 3 \rangle$ and $\vec{b} = \langle 3, 2, 1 \rangle$.

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \sin\theta$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 2 & 1 \end{vmatrix} = \langle -7, 7, 7 \rangle$$

$$|\vec{a} \times \vec{b}| = \sqrt{49 + 49 + \underset{49}{\cancel{44}}} =$$

$$= 7 \cdot \sqrt{3}$$

$|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{14}$

$$\therefore \sin\theta = \frac{9\sqrt{2}}{\sqrt{14}\sqrt{14}} = \frac{9\sqrt{2}}{14}$$

$$= \frac{7\sqrt{3}}{14} \doteq \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

Ex. 6 Sketch

$$z - x^2 + y^2 + 4x + 6 = 0$$

$$z - (x-2)^2 + y^2 - 6 = -4$$

$$z - (x-2)^2 - y^2 = 2$$

This is a paraboloid

$$Z = 2 + X^2 - Y^2$$

Curves down in y,

Curves up in x.

$\therefore \ \vec{r} = \langle 0, 1, 0 \rangle + t\langle 1, -2, 1 \rangle$

$x = t, \quad y = 1 - 2t, \quad z = t$

Ex. 2. Find the plane that contains

$P(1, 2, 3) \quad Q(2, -1, 4) \quad R(4, 1, 0)$

$\overrightarrow{PQ} = \langle 1, -3, 1 \rangle \quad \overrightarrow{PR} = \langle 3, -1, -3 \rangle$

$\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are both

tangent to plane.

$$\therefore \quad \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 1 \\ 3 & -1 & -3 \end{vmatrix}$$

$$= 10\vec{i} + 6\vec{j} + 8\vec{k}$$

or $\quad 5\vec{i} + 3\vec{j} + 4\vec{k}$

Note that $(1, 2, 3)$ is in plane.

$$5(x-1) + 3(y-2) + 4(z-3) = 0.$$

or: $5x + 3y + 4z = 23$