12.6 Surfaces in 3-dimensions

The cylinder is described by

$$x^2 + y^2 = r^2$$

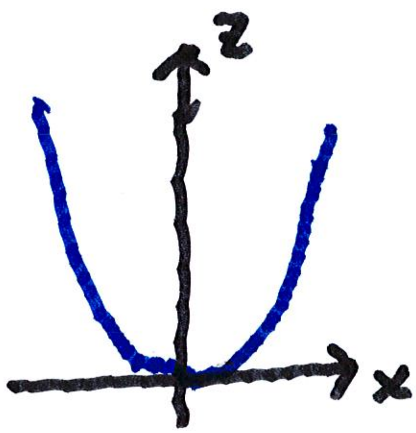
Note that the equation is independent of z.

Thus z can assume any value.

Ex. Sketch the surface

(ind. of y)

$z = x^2$. First draw the

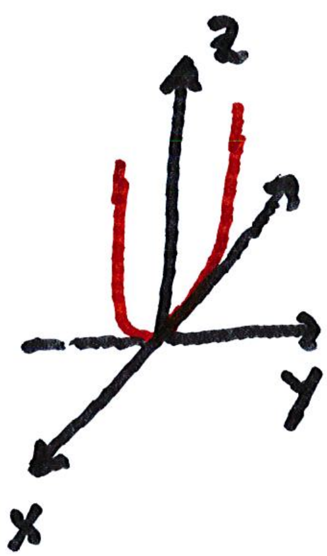


curve in the

xz-plane.

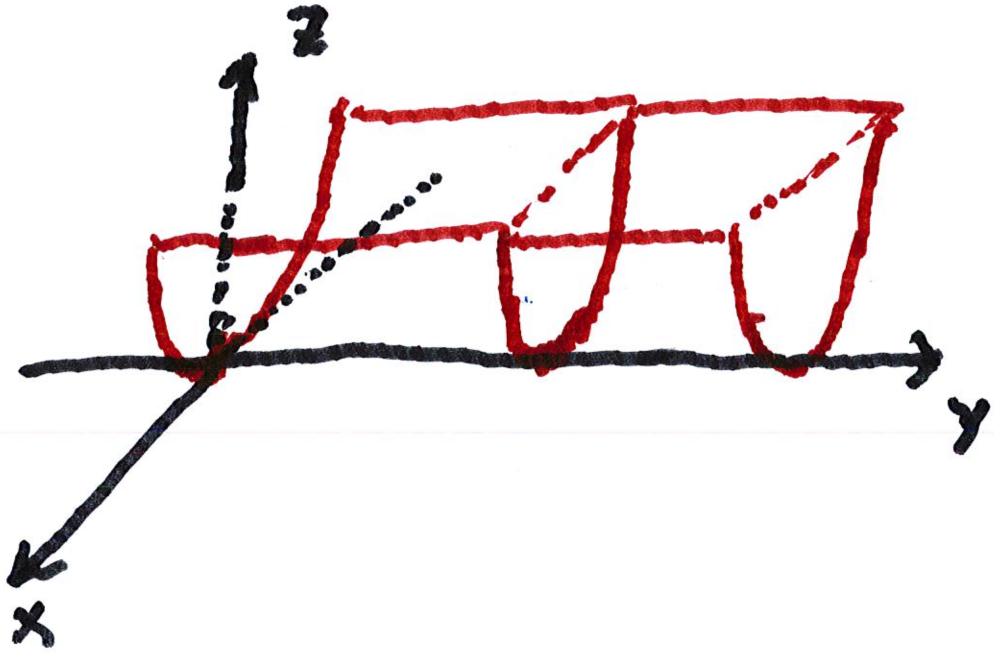
Then draw curve

in the xz-plane in 3 dim.



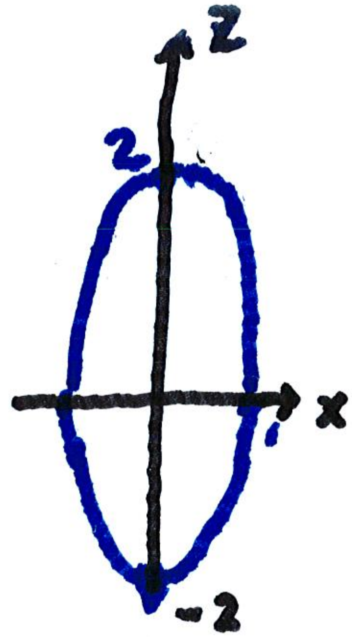
Then slide the

curve in y-direction



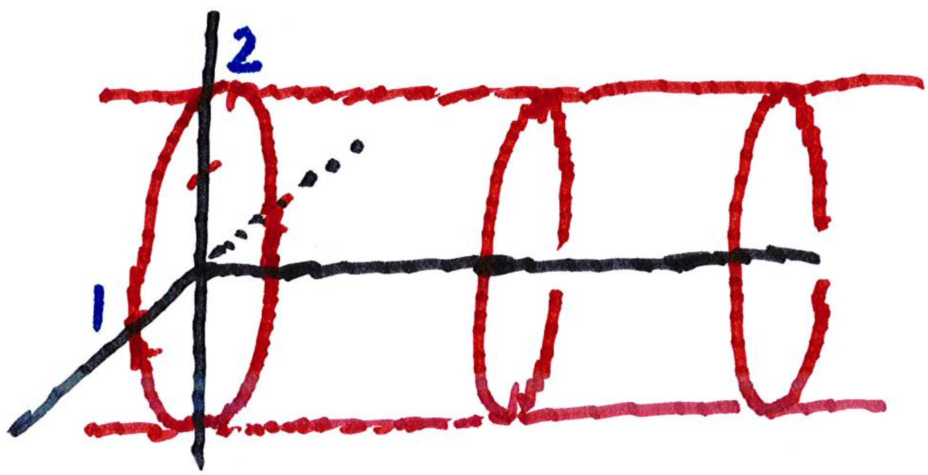
Ex Sketch the cylinder

$$x^2 + \frac{z^2}{4} = 1$$



The equation is independent of y.

Now slide the ellipse in the y- direction



Quadric Surfaces.

This is the graph of second-degree equation satisfying

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

By making translations and rotations, this can be put in the form

I $\quad Ax^2 + By^2 + Cz^2 + J = 0$ or

II $\quad Ax^2 + By^2 + Iz = 0$

Ex. Suppose in type I that

A, B, and C are all positive.

Ex. $\quad x^2 + \frac{y^2}{4} + \frac{z^2}{16} = 1$.

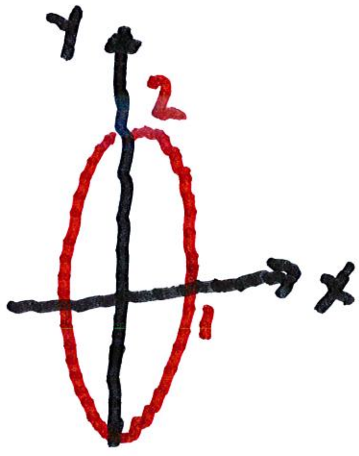
This is an ellipsoid.

We can write this as

$$x^2 + \frac{y^2}{4} = 1 - \frac{z^2}{16}$$

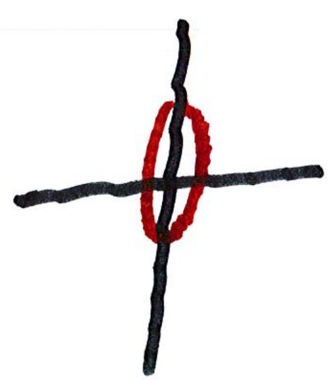
When $z = 0$, the "trace"

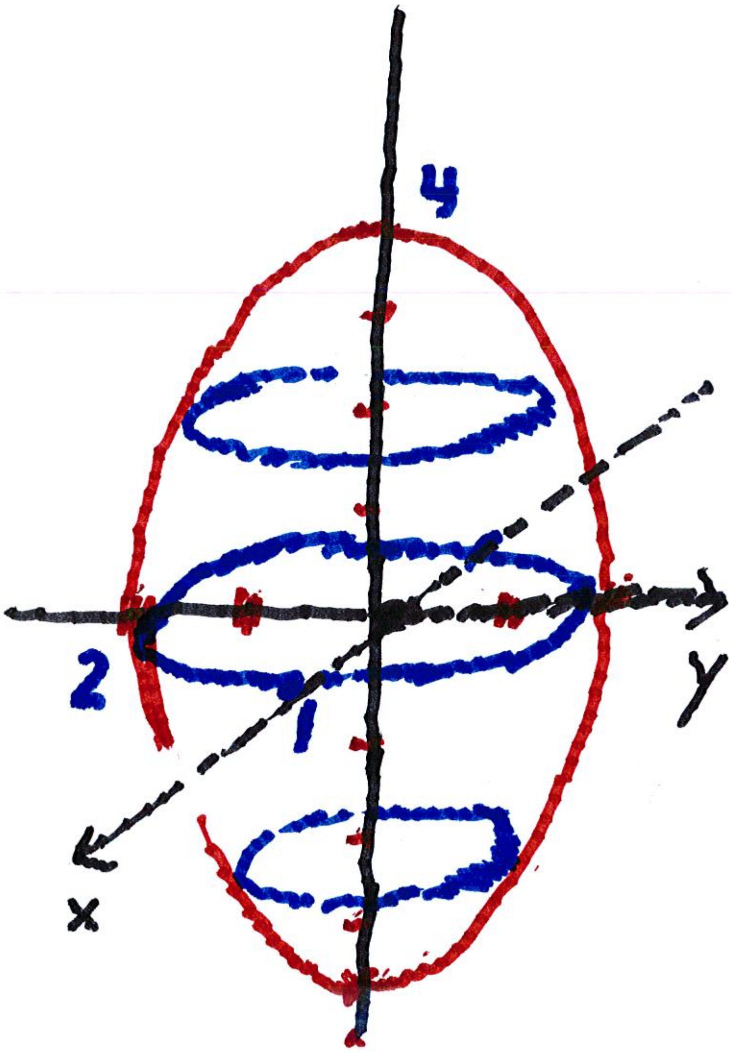
is $\frac{x^2}{1} + \frac{y^2}{4} = 1$



As z increases

$1 - \frac{z^2}{16}$ decreases





The ellipses are traces for different values of z ($-4 \le z \le 4$)

Ex. Now consider the elliptic paraboloid.

$$z = x^2 + \frac{y^2}{4}$$

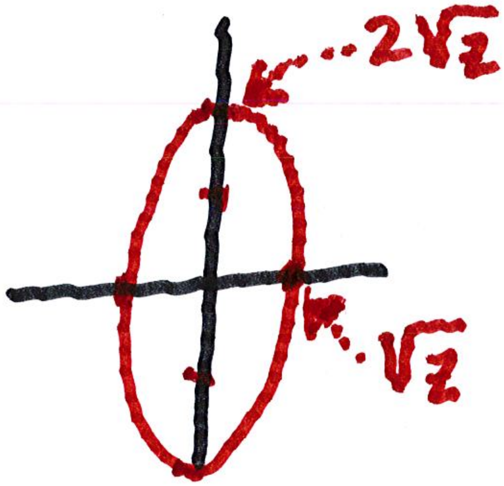
If $z = 0$, then $x = 0$, $y = 0$

If $z > 0$,

$$(\sqrt{z})^2 = x^2 + \frac{y^2}{4}$$

$$\rightarrow \quad 1 = \left(\frac{x}{\sqrt{z}}\right)^2 + \left(\frac{y}{2\sqrt{z}}\right)^2$$

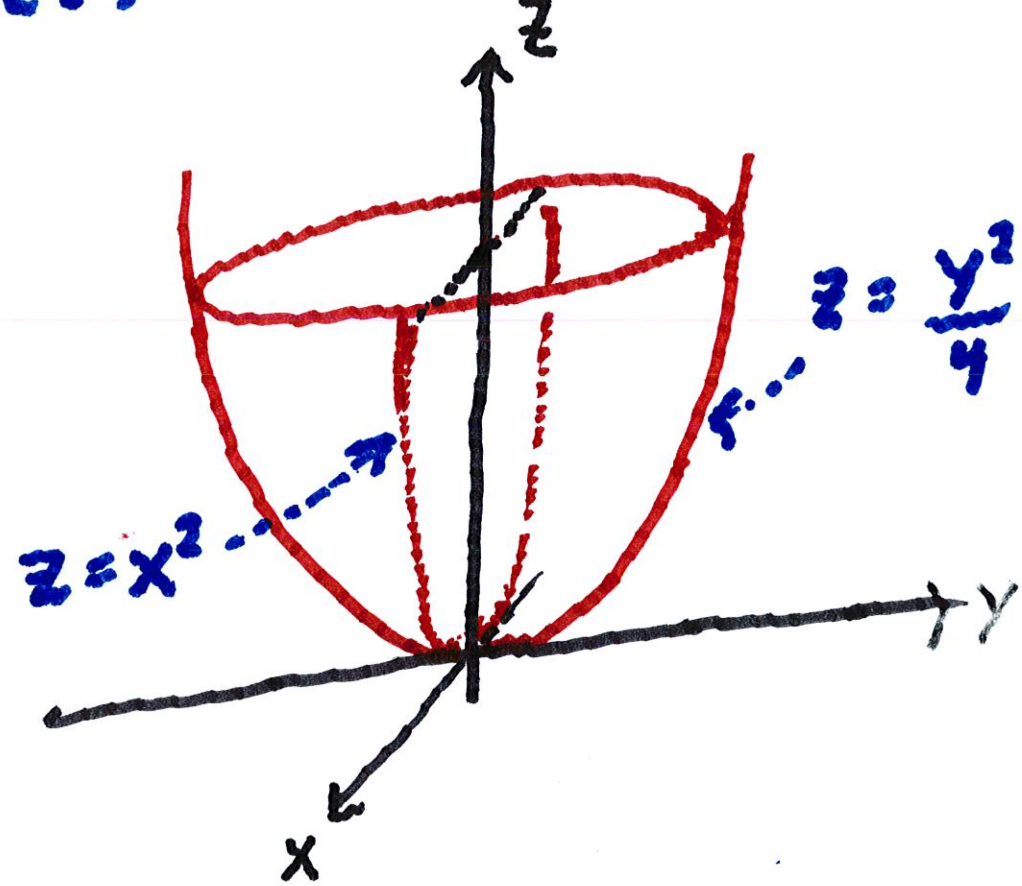
This is an ellipse



, which gets bigger as $\sqrt{z}$ increases.

If $y=0$, the trace is $z=x^2$, and if $x=0$, the trace is $z=\dfrac{y^2}{4}$

We get:



Ex. The hyperbolic paraboloid

is given by $z = y^2 - x^2$:

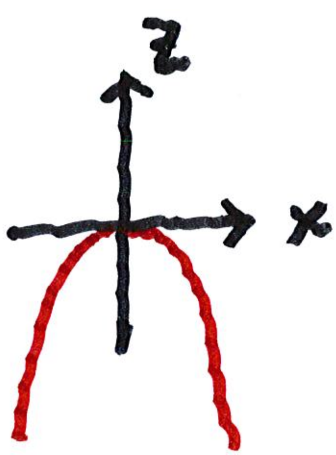
For fixed y, $z = y^2 - x^2$

(if $y = 0$) $z = -x^2 + y^2$

Ex. The hyperbolic paraboloid is given by

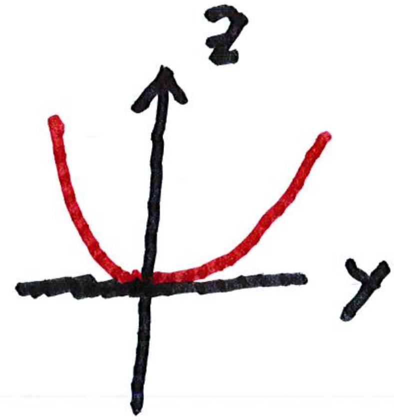
$z = y^2 - x^2$

For $y = 0$ the surface is given by $z = -x^2$

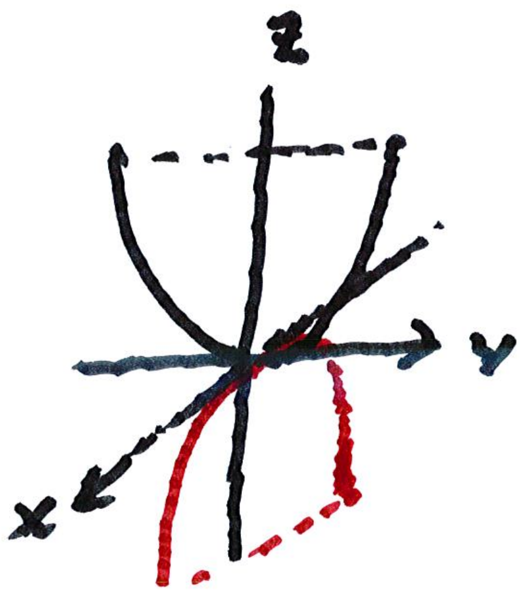


and if $x = 0$,

and the trace of $x = 0$ is $z = y^2$

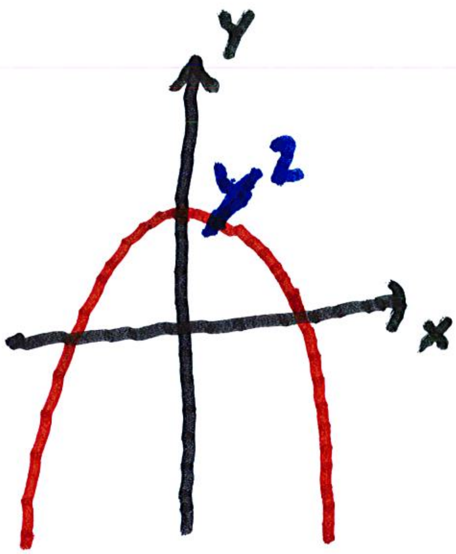


In 3 dimensions:



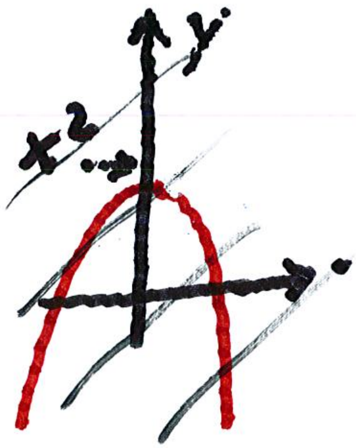
More generally, for fixed y,

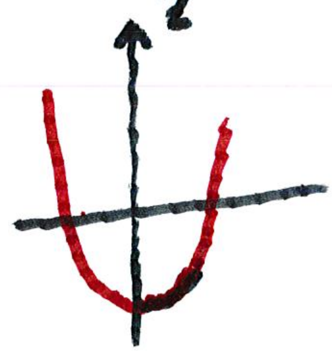
the curve is $z = y^2 - x^2$

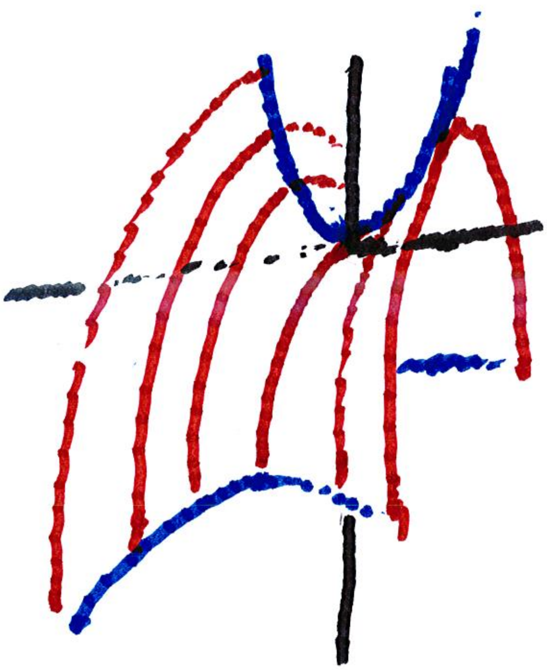


The general surface is :

For fixed x, $z = y^2 - x^2$







The origin $(0,0,0)$ is a saddle point

Ex. Now consider the

equation

$$x^2 + y^2 - z^2 = \underset{<0}{-4}$$

or $x^2 + y^2 = z^2 - 4$

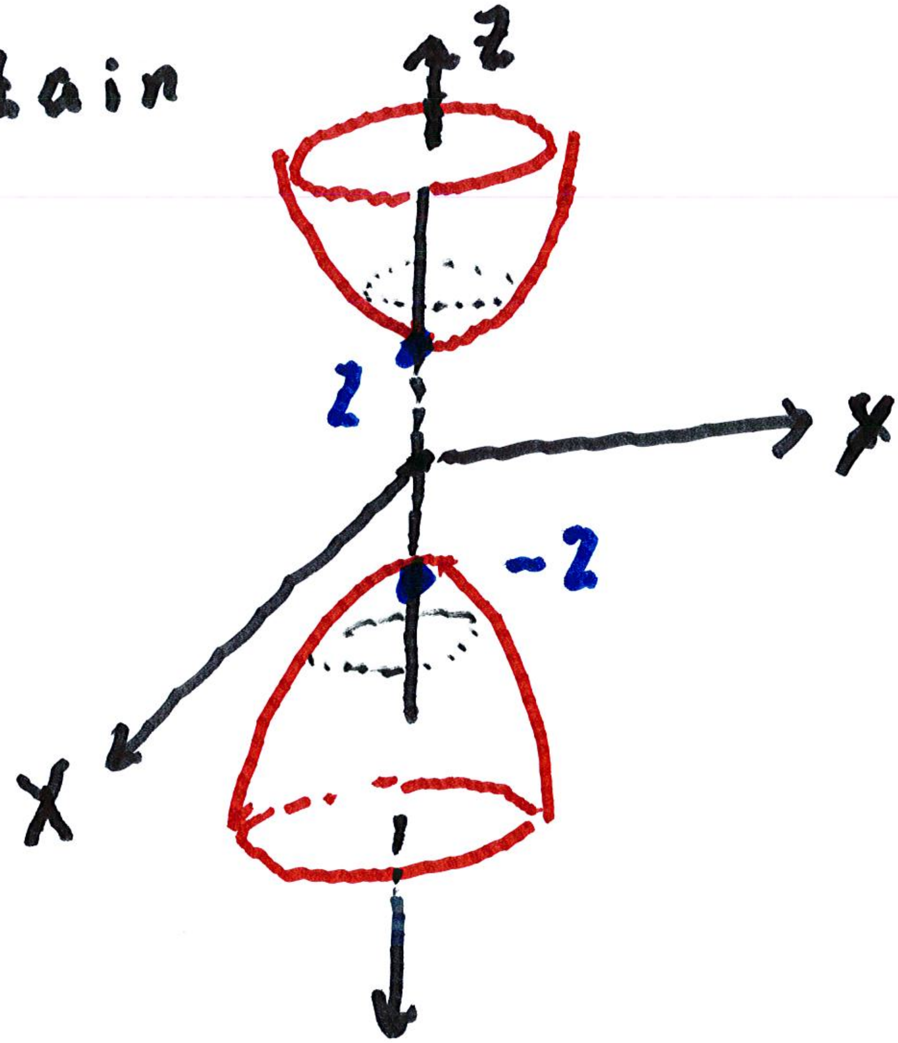
There is no solution

if $z^2 < 4$. If $z^2 \geq 4$

we have a circle of

radius of $\sqrt{z^2-4}$.

We obtain



This is a "hyperboloid

of 2 sheets

Now suppose that

2 of the coefficients

A, B, C are positive

and the other is negative.

$$> 0$$

Ex. $x^2 + y^2 - z^2 = 4$

or $x^2 + y^2 = 4 + z^2$

For fixed z, the

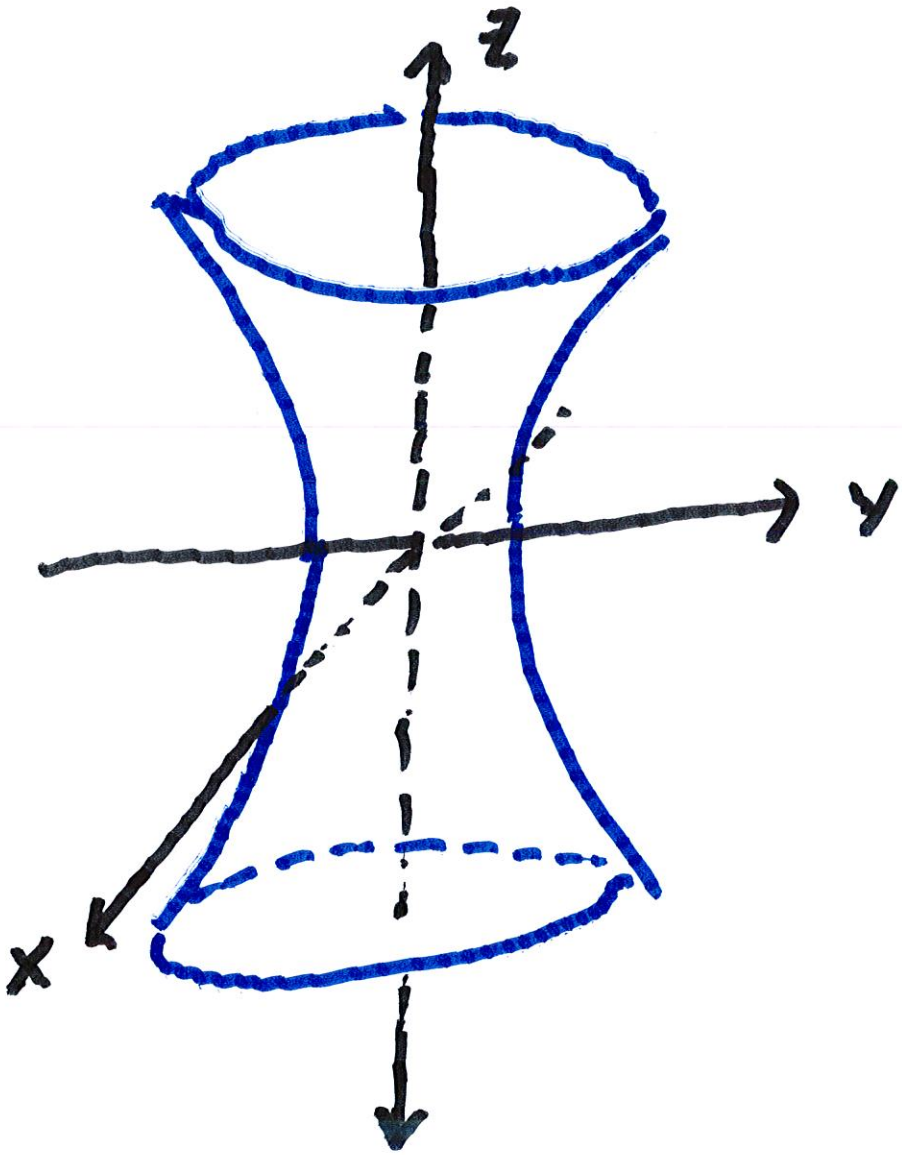
trace is

$$x^2 + y^2 = \left(\sqrt{4+z^2}\right)^2$$

This is a circle of

radius $\sqrt{4+z^2} \to \infty$ as

$z, -z \to \infty$



This is called by a

"hyperboloid of 1 sheet"

Ex. Now suppose that

$$x^2 + y^2 - z^2 = 0$$

or $x^2 + y^2 = z^2 = |z|^2$

This is a circle of radius $|z|$. We obtain a cone