Ex 1. Find the equation of the line L that is the intersection of

$$\underbrace{x + y + z = 1}_{P_1} \quad \text{and} \quad \underbrace{x + 2y + 3z = 2}_{P_2}$$

The line L lies in P_1, so its direction vector is $\perp \vec{n}_1$

Similarly the same vector

is $\perp$ to the normal $\vec{n}_2$

$$\therefore \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\vec{v} = \vec{i} - 2\vec{j} + \vec{k}$$

To find P in both planes

set $x_3 = 0$

$$\rightarrow \left.\begin{aligned} x + y &= 1 \\ x + 2y &= 2 \end{aligned}\right\} \rightarrow y = 1, \quad x = 0$$

$\therefore \vec{r} = \langle 0, 1, 0 \rangle + t\langle 1, -2, 1 \rangle$

$x = t, \quad y = 1 - 2t \quad z = t$

Ex. 2. Find the plane that contains

$P(1, 2, 3) \quad Q(2, -1, 4) \quad R(4, 1, 0)$

$\overrightarrow{PQ} = \langle 1, -3, 1 \rangle \quad \overrightarrow{PR} = \langle 3, -1, -3 \rangle$

$\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are both tangent to plane.

$$\therefore \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 1 \\ 3 & -1 & -3 \end{vmatrix}$$

$$= 10\vec{i} + 6\vec{j} + 8\vec{k}$$

or $5\vec{i} + 3\vec{j} + 4\vec{k}$

$P = (1, 2, 3)$ is in plane

$$\therefore \quad (5, 3, 4) \cdot (x-1, y-2, z-3) = 0$$

or $\quad 5x + 3y + 4z = 23$

Ex. 3. Sketch

$$x^2 - y^2 + z^2 - 4x - 2y - 2z + \overset{C}{4} = 0$$

Comp. the square:

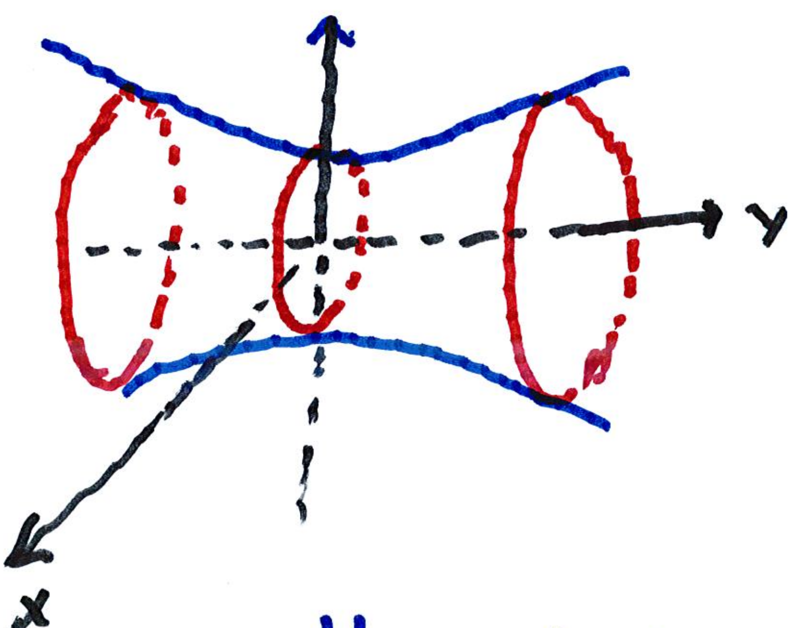
$$(x-2)^2 - (y+1)^2 + (z-1)^2 = \cancel{6-4=2}$$

$$4 - 4 = 0$$

Look at

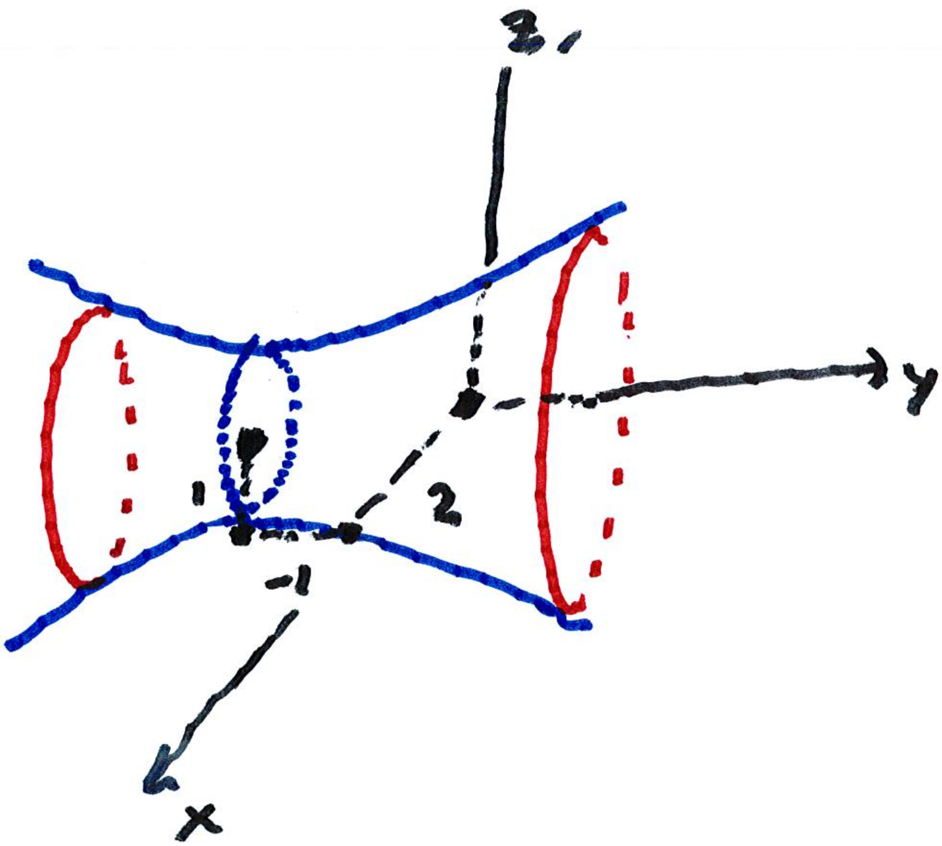
$$X^2 - Y^2 + Z^2 = 2$$

$$X^2 + Z^2 = 2 + Y^2$$



Hyperboloid of 1 sheet

Recall center is at $(2,-1,1)$



What if c had been $= 6$

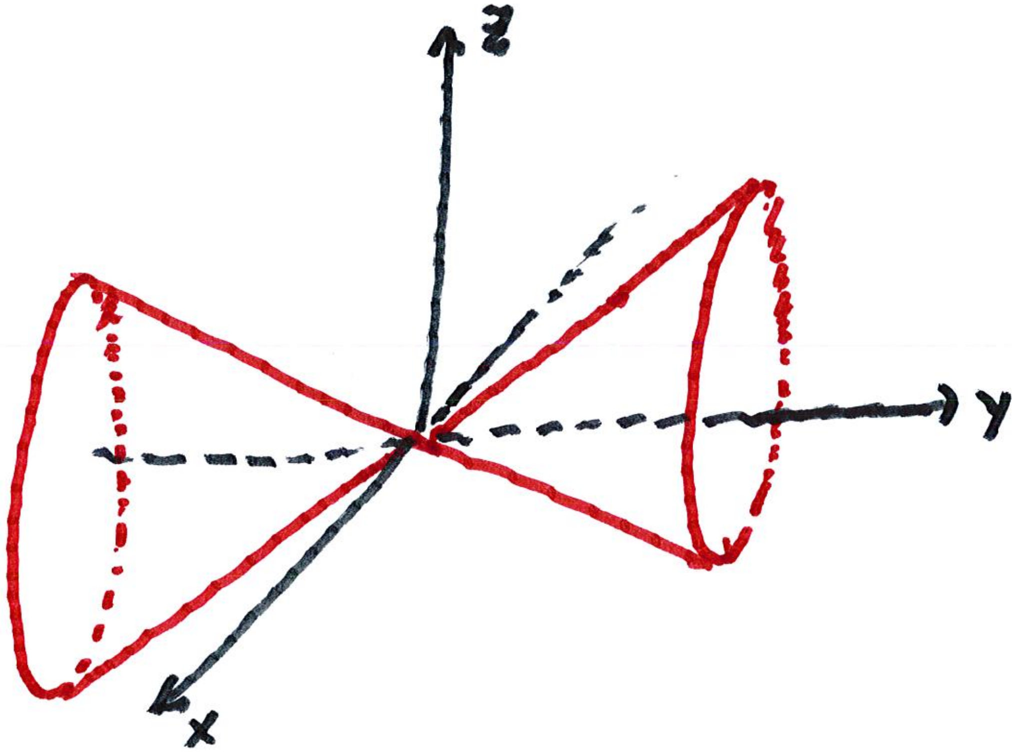
$$\Rightarrow (x-2)^2 - (y+1)^2 + (z-1)^2 = 6-6 = 0$$

or $\quad X^2 - Y^2 + Z^2 = 0$

or $\quad X^2 + Z^2 = Y^2$ (a cone)

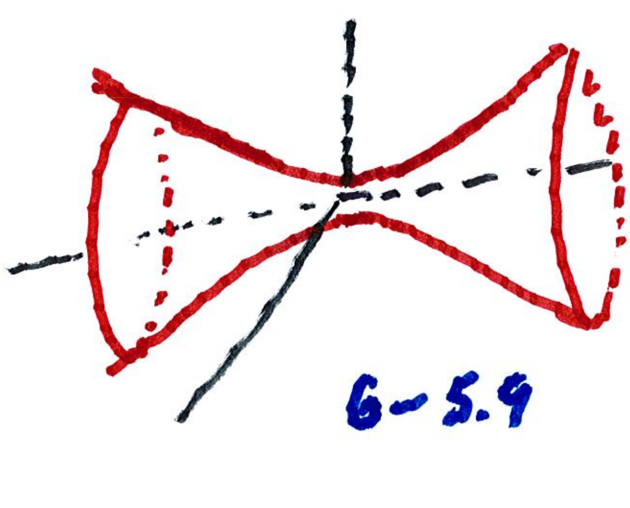
For fixed Y, we get a circle

of radius $|Y|$

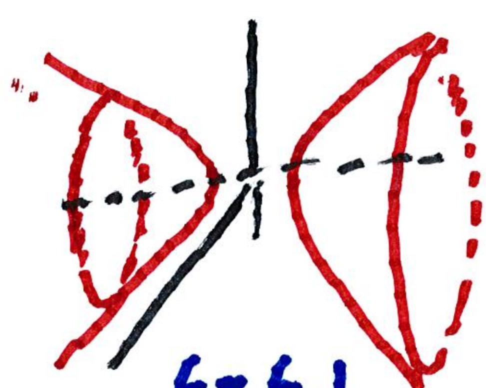


If C had been 5.9, then

$$(x-2)^2 - (y+1)^2 + (z-1)^2 = 6 - 5.9$$



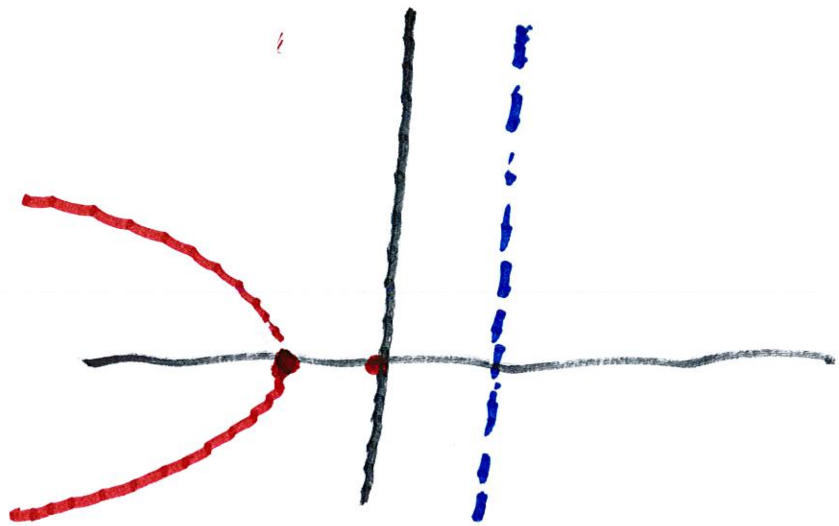
$6 - 5.9$



$6 - 6.1$

Ex. 4. Find an equation for the surface consisting of all points that are equidistant from the point $(-1, 0, 0)$ and the plane $x = 1$.

First look on xy-plane $(z = 0)$



$p = 1$ $\qquad -4px = y^2$

or $\quad -4x = y^2$

If we rotate about the x-axis

we get $\quad \underline{-4x = y^2 + z^2}$

Ex 3 Find the saddle point of the surface

$$4x^2 - z^2 + 8x - 2z + 4y = 4$$

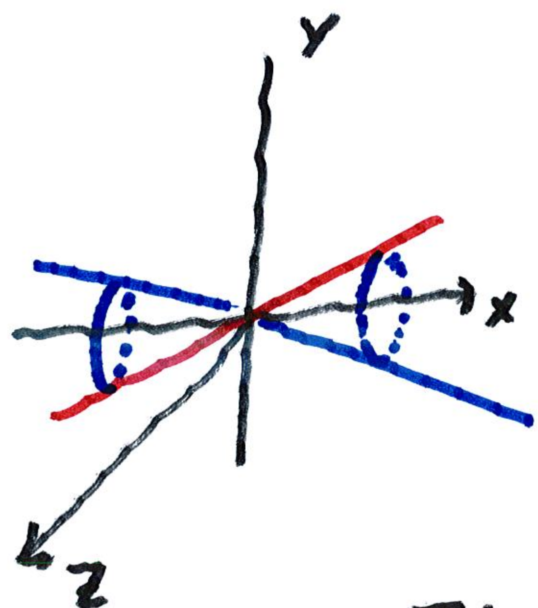
$$4(x+1)^2 - (z+1)^2 + 4y = 7$$

$$\cancel{4(y-z)} =$$

$$y = \frac{(z+1)^2}{4} - (x+1)^2 + \frac{7}{4}$$

$x = -1$, $z = -1$, $y = \frac{7}{4}$

Ex.6 Find an equation for the surface obtained by rotating the line $x = 3y$ about the x-axis



$x = 3y$

$\rightarrow y = \frac{x}{3}$

$y^2 = \frac{x^2}{9}$

$\rightarrow y^2 + z^2 = \frac{x^2}{9}$

This is a cone.

Ex.7 Sketch the plane by using intercepts:

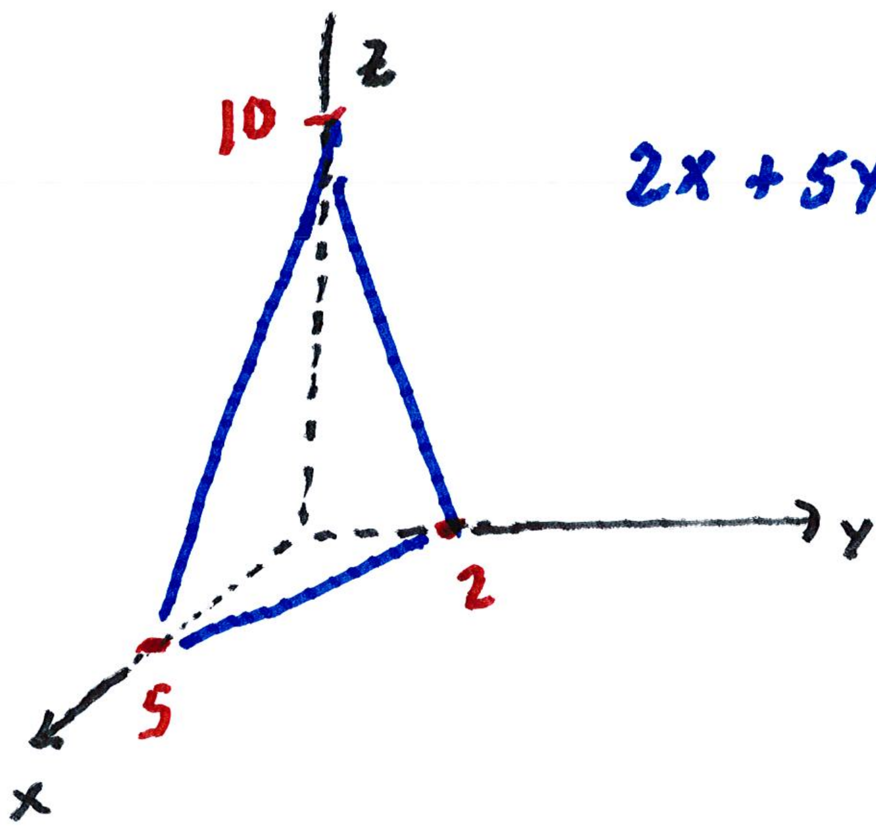
$2x + 5y + z = 10$

$y, z = 0 \Rightarrow x = 5 \quad (5, 0, 0)$

$x, z = 0 \rightarrow 5y = 10 \rightarrow y = 2$

$(0, 2, 0)$

$xy = 0 \rightarrow z = 10 \rightarrow (0, 0, 10)$



$$2x + 5y + z = 10$$

Ex.8 A force is given by a vector $\vec{F} = 3\vec{i} + 4\vec{j} + 5\vec{k}$ and moves a particle from the point $P(2, 1, 0)$ to $Q(4, 6, 2)$. Find the work done.

$\overrightarrow{PQ} = \langle 2, 5, 2 \rangle \qquad W = \vec{F} \cdot \vec{D}$

$W = \langle 3, 4, 5 \rangle \cdot \langle 2, 5, 2 \rangle$

$= 6 + 20 + 10 = 36$

Ex 9 Find the scalar projection

and vector projection of

$\vec{b} = \langle 1, 4 \rangle$ onto $\vec{a} = \langle 3, 2 \rangle$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|a|^2} \vec{a}$$

$$= \frac{11}{13} \langle 3, 2 \rangle$$

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scalar eq'n is $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \dfrac{11}{\sqrt{13}}$

Ex 10. Find the sine of the angle between $\vec{v} = \langle 2, -1, 3 \rangle$ and $\vec{w} = \langle 3, 2, 1 \rangle$.

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \sin\theta$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 2 & 1 \end{vmatrix} = \langle -7, 7, 7 \rangle$$

$$|\vec{a} \times \vec{b}| = \sqrt{49 + 49 + \underset{49}{\cancel{64}}} = \sqrt{162} = 9\sqrt{2}$$

$$7 \cdot \sqrt{3}$$

$|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{14}$

$\therefore \sin\theta = \dfrac{9\sqrt{2}}{\sqrt{14}\sqrt{14}} = \dfrac{9\sqrt{2}}{14}$

$= \dfrac{7\sqrt{3}}{14} \doteqdot \dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{\pi}{3}$