

13.1 Vector Functions

A vector function

assigns a vector

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

to each t in the domain of \vec{r} . Usually the domain is the set of t such that $f(t)$, $g(t)$, and $h(t)$ are all defined.

Ex. Define

$$\vec{r}(t) = \left\langle \sqrt{t+1}, \ln(2-t), \frac{1}{t^2} \right\rangle$$

Find the domain of \vec{r} .

t must satisfy

$$t+1 \geq 0, \quad 2-t > 0, \quad \text{and } t \neq 0$$

$$\text{or } t \geq -1, \quad t < 2, \quad \text{and } t \neq 0$$

$$\therefore \text{Dom}(\vec{r}) = \left\{ -1 \leq t < 2, \quad t \neq 0 \right\}$$

We define

$$\lim_{t \rightarrow a} \langle f(t), g(t), h(t) \rangle$$

$$= \left\langle \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \right\rangle,$$

(provided that these
3 limits exist).

Ex. Compute

$$\lim_{t \rightarrow 0^+} \left(t \ln t, \sqrt[3]{t}, 2t+3 \right)$$

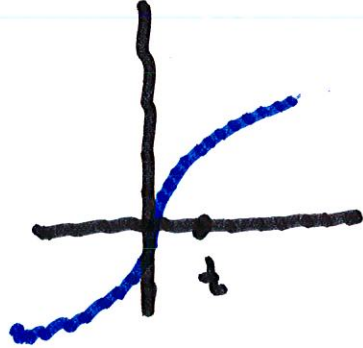
$$\lim_{t \rightarrow 0^+} t \ln t = \lim_{t \rightarrow 0^+} \frac{\ln t}{\frac{1}{t}}$$

$$\stackrel{\text{L'Hop}}{=} \lim_{t \rightarrow 0^+} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t \rightarrow 0^+} (-t)$$

$$= 0$$



$$\lim_{t \rightarrow 0^+} \sqrt[3]{t} = 0$$



$$\lim_{t \rightarrow 0^+} (2t + 3) = 2 \cdot 0 + 3 = 3$$

$$\therefore \text{limit} = (0 + 0 + 3 = 3)$$

3

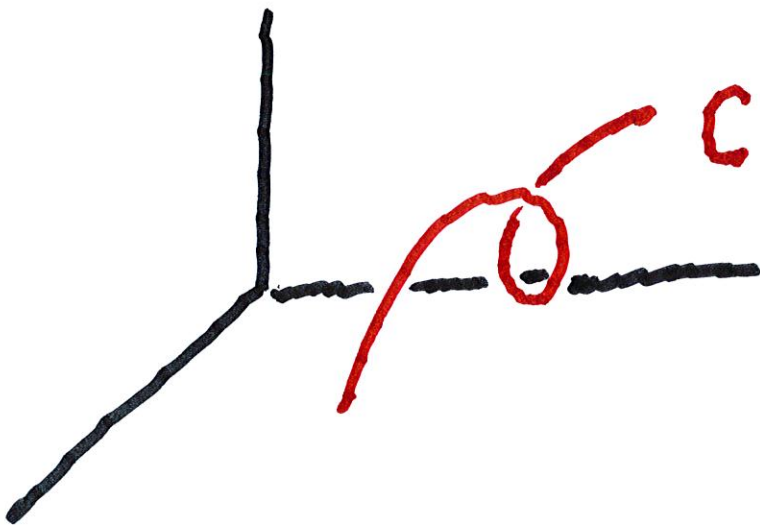
A function $\vec{r}(t)$ is
continuous at a if

$$\lim_{t \rightarrow a} \vec{r}(t) = \vec{r}(a).$$

Suppose that $f(t)$, $g(t)$
and $h(t)$ are all continuous
at all t in an interval I .

We say that a set C of points $(x, y, z) \in \mathbb{R}^3$ is a space curve if for each $(x, y, z) \in C$ there is a number t such that

$$x = f(t), \quad y = g(t), \quad \text{and} \quad z = h(t).$$



A line segment S is a space curve. Let $\pi_0 = (x_0, y_0, z_0)$

and $\pi_1 = (x_1, y_1, z_1)$.

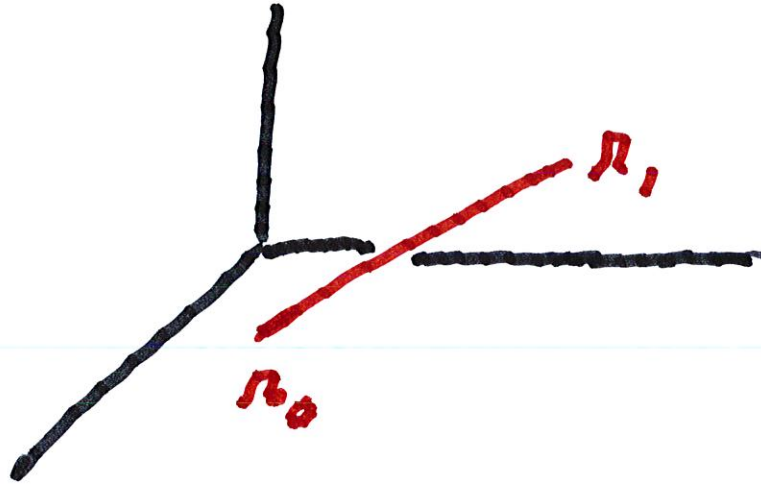
Define $\vec{v} = \overrightarrow{\pi_0 \pi_1}$,

Let $\vec{r}(t) = \vec{\pi}_0 + t\vec{v}$, for $0 \leq t \leq 1$.

$$\vec{r}(t) = \vec{\pi}_0 + t(\vec{\pi}_1 - \vec{\pi}_0)$$

$$\vec{r}(0) = \vec{\pi}_0 \text{ and } \vec{r}(1) = \vec{\pi}_0 + (\vec{\pi}_1 - \vec{\pi}_0)$$

$$= \vec{\pi}_1$$



$$\therefore \vec{\pi}(t) = \vec{\pi}_0 + t(\vec{\pi}_1 - \vec{\pi}_0)$$

Ex. Parameterize the

segment between $(2, 3, 1)$

and $(-1, 9, 10)$

$$\vec{v} = \{n_1 - n_0\}$$

$$= \{-1, 9, 10\} - \{2, 3, 1\}$$

$$= \{-3, 6, 9\}$$

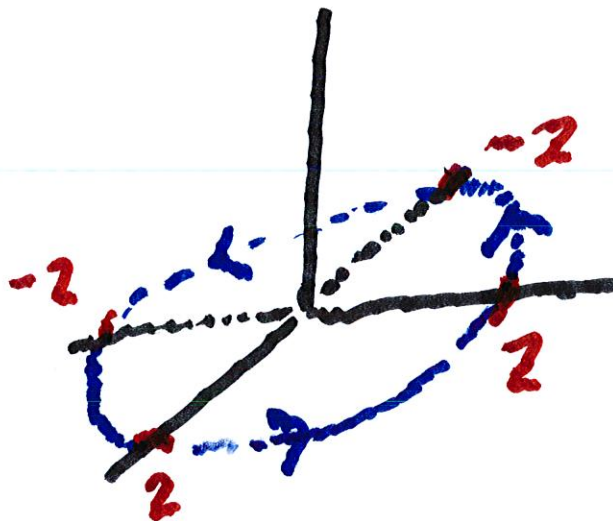
$$\therefore \vec{r}(t) = \{2, 3, 1\} + t\{-3, 6, 9\}$$

Ex. Sketch the curve

$$x = 2 \cos t \quad y = 2 \sin t, \quad z = t$$

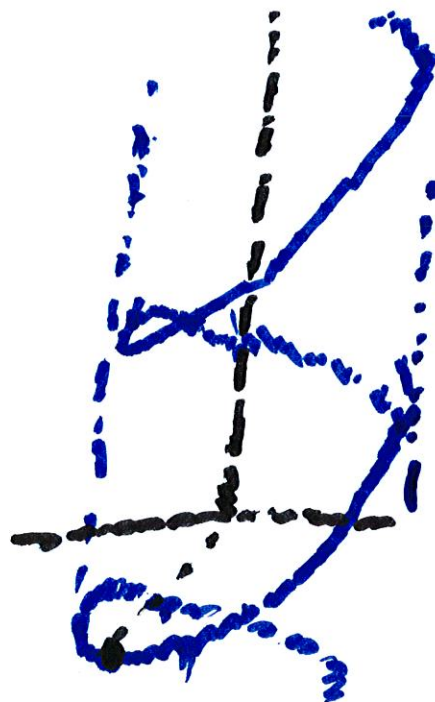
First suppose that

$$Z(t) = 0$$



$(2 \cos t, 2 \sin t)$ goes in
a circle of radius 2.

If $Z(t) = t$



We get a helix that winds up around the z -axis.

Ex. Find a parametric curve described by

$$x^2 + 4y^2 = 4 \quad \text{and} \quad z = x^2$$



Divide by 4

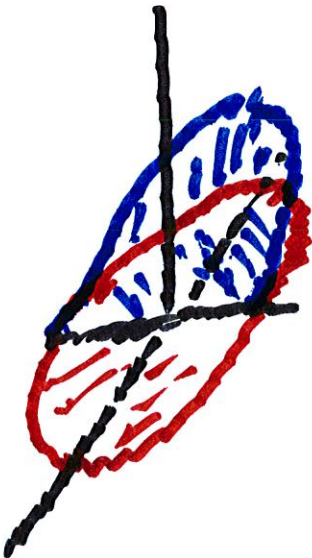
$$\frac{x^2}{4} + y^2 = 1 \quad \text{and} \quad z = x^2$$

Left Curve



$$\text{Set } x = 2 \cos t \rightarrow z = x^2$$

$$\text{and } y = \sin t \quad z = 4 \cos^2 t$$



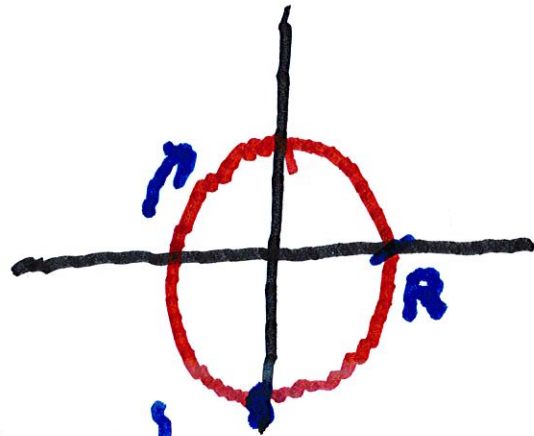
• 4

Cycloid (Rolling Wheel)

Imagine a point on a

tire of radius R

at $\theta = \frac{3\pi}{2}$



$$x = R \cos \left(\frac{3\pi}{2} - t \right)$$

$$y = R \sin \left(\frac{3\pi}{2} - t \right)$$

Use formulas for

$\cos(A-B)$ and $\sin(A-B)$:

we get $x = -R \sin t$

and $y = -R \cos t$

If wheel rests on x -axis

$$x = -R \sin t$$

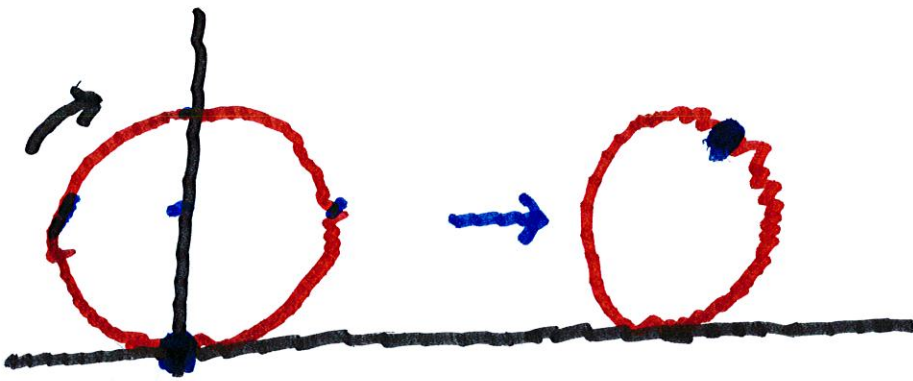
$$y = R - R \cos t$$

After t seconds, the wheel moves Rt to the

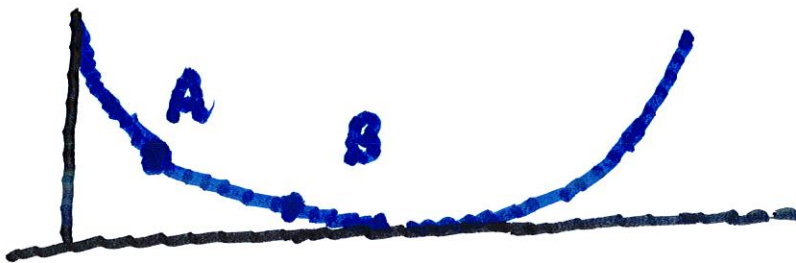
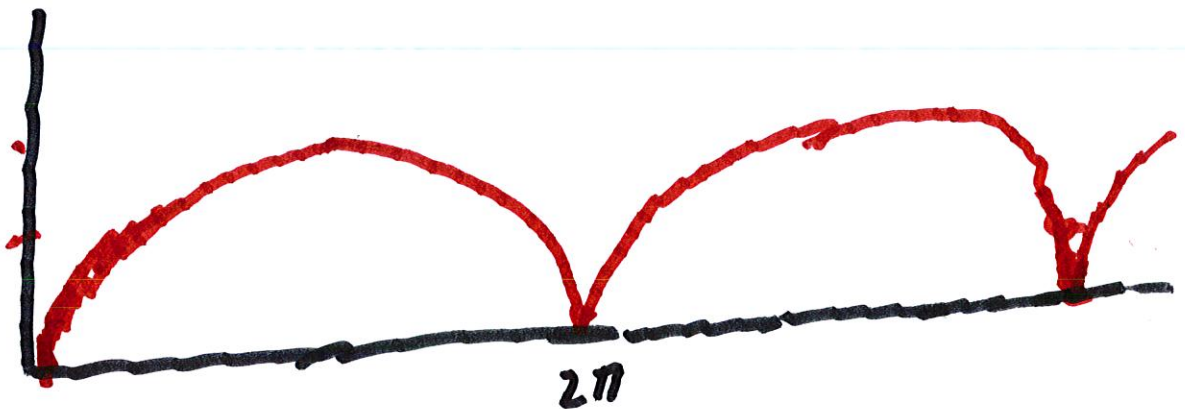
right:

$$x = Rt - R \sin t$$

$$y = R - R \cos t$$



Location of Blue Spot (Cycloid)



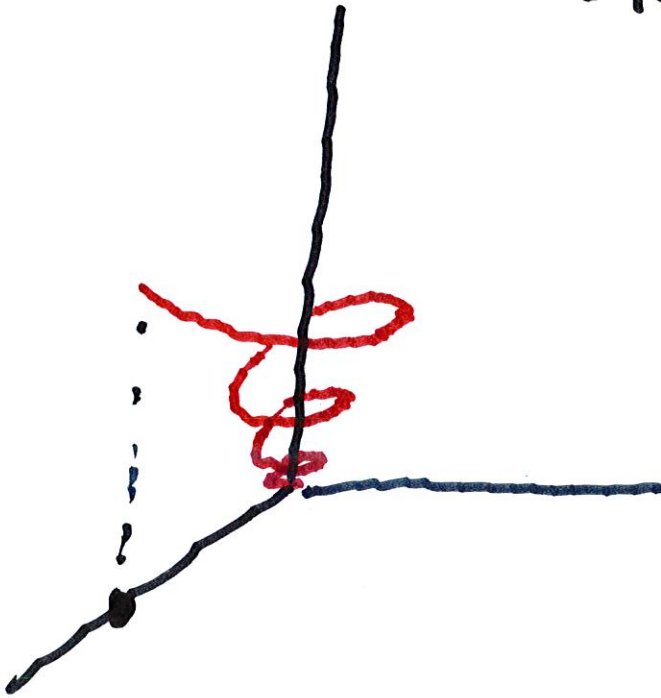
Among all curves joining
A and B, the cycloid takes
the least time. (Bernoulli)

Ex. Sketch $x = e^{-t} \cos 10t$

$$y = e^{-t} \sin 10t$$

$$z = t, \quad t > 0$$

$$= 1-t$$



Ex. Two particles have

trajectories given by

$$\vec{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle$$

$$\vec{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle$$

Looking at x and z are equal

in $\vec{r}_1(t)$, it must be that

$$4t - 3 = 5t - 6$$

$$\therefore t = 3$$

$$\vec{r}_1(3) = \langle 9, 9, 9 \rangle$$

Also

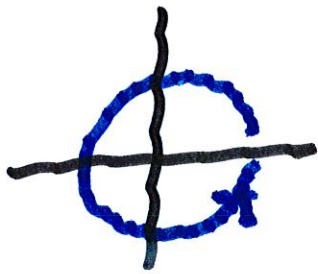
$$\vec{r}_2(3) = \langle 9, 9, 9 \rangle$$

They do collide !!!

Ex. Find a vector function

that represents:

$$x^2 + y^2 = 4, \quad z = xy$$



$$x = 2 \cos t, \quad y = 2 \sin t$$

$$\therefore z(t) = 4 \cos t \sin t$$