Quiz 1 Math 341 **Name** _____

Prove that $3 + 11 + \ldots + (8n - 5) = 4n^2 - n$ for all $n \in N$

Solution

To prove by induction, we must
1. Show it is true for $n = 1$
2. Assume true for k and prove it is true for $k + 1$.

For $n = 1$
$8(1) - 5 = 3$ and $4(1)^2 - 1 = 3$
Thus it is true for $n = 1$

Assume true for k and prove true for $k + 1$

Simplify the left hand side
$3 + 11 + \ldots + (8k - 5) + (8(k + 1) - 5) =$
$3 + 11 + \ldots + 8k - 5 + 8k + 8 - 5$
Applying the formula for k, the left hand side becomes
$[3 + 11 + \ldots + 8k - 5] + 8k + 8 - 5 =$
$[4k^2 - k] + 8k + 3 = 4k^2 + 7k + 3$

Simplify the right hand side
$4(k + 1)^2 - (k + 1) =$
$4(k^2 + 2k + 1) - k - 1 =$
$4k^2 + 8k + 4 - k - 1 =$
$4k^2 + 7k + 3$
Thus it is true for $k + 1$

By induction, the formula is true for all $n \in N$.