Quiz 5 MA341 **Name** ______________________________
Spring 2018

Let f, g be defined on $A \subset \mathbb{R}$ to $\mathbb{R}$ and let c be a cluster point of A. Suppose that f is bounded on a neighborhood of c and that $\lim_{x\to c} g = 0$.

Prove that $\lim_{x\to c} fg = 0$.

Solution

Since f is bounded, it follows that there is a constant $M > 0$ and a constant $\delta_1 > 0$ so that $|f(x) \le M|$ for all $x \in A$ with $0 < |x - c| < \delta_1$.

Since $\lim_{x\to c} g = 0$, it follows that there is a $\delta_2 > 0$ so that $|g(x) - 0| < \frac{\epsilon}{M}$ when $x \in A$ and $0 < |x - c| < \delta_2$.

Let $\delta = \min\{\delta_1, \delta_2\}$. Then

$|f(x)g(x) - 0| = |f(x)g(x)| \le M\frac{\epsilon}{M} = \epsilon$ when $x \in A$ and $0 < |x - c| < \delta$.

Thus, $\lim_{x\to c} f(x)g(x) = 0$.