Prove that $5^{2n} - 1$ is divisible by 8 for all $n \in N$.

Solution

To prove by induction, we must
1. Show it is true for $n = 1$
2. Assume true for n and prove it is true for $n + 1$.

Let $P(n)$ be the statement $5^{2n} - 1$ is divisible by 8.

When $n = 1$, $5^2 - 1 = 24$, which is divisible by 8.
This proves $P(1)$.

Suppose that $P(n)$ is true, i.e., $5^{2n} - 1$ is divisible by 8.

Then $5^{2(n+1)} - 1 = 5^2 \cdot 5^{2n} - 1 = 5^2(5^{2n} - 1) + (5^2 - 1)$.

Note that $5^{2n} - 1$ is divisible by 8 using $P(n)$ and also $5^2 - 1 = 24$ is divisible by 8. Thus we have proved that $P(n+1)$ is true.

We conclude that $P(n)$ is true for all $n \in N$.