

Math 341 Spring 2018 Study Guide for Exam 2

The exam covers Lecture 14 through Lecture 28. This includes section 4.2 through section 6.3.

Statements of theorems from the textbook to memorize

1. Maximum-Minimum Theorem, p. 136
2. Mean Value Theorem, p. 173

Review problems

1. Use the Mean Value Theorem to prove that if $f : [a, b] \rightarrow \mathbb{R}$ satisfies $f'(x) = 0$ for all $x \in [a, b]$, then $f(x)$ is a constant for all $x \in [a, b]$.
2. Let $f(x) = \sqrt{x}$ for $0 \leq x < \infty$. Show f is continuous at 0.
3. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ and that $g : \mathbb{R} \rightarrow \mathbb{R}$. If f and g are both continuous on \mathbb{R} , show that $g \circ f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous.
4. Show that $f(x) = \sqrt{x}$ is Lipschitz on $[a, \infty)$ for any $a > 0$.
5. Show if f is differentiable at x_0 , then f is continuous at x_0 .

6. Evaluate the following limits

(a) $\lim_{x \rightarrow \infty} \frac{\ln x}{x^2}$

(b) $\lim_{x \rightarrow 0^+} x^{2x}$

7. Show that the function $f(x) = \frac{1}{x}$ is uniformly continuous on $[1, \infty)$.

8. Use the Location of Roots Theorem to show that there is a number $c \in (0, \frac{\pi}{2})$ that is a root of the equation $x^2 - \cos x = 0$.

9. Use the limit laws and composition laws to show that $\frac{\sqrt{2x+\sqrt{x}}}{3+x}$ is continuous on $[0, \infty)$. You must justify your reasoning.

10. (a) What does it mean to say that a function f on a set A is uniformly continuous?

(b) Show that $f(x) = x^2$ on \mathbb{R} is NOT uniformly continuous by using the sequences $x_n = n + \frac{1}{n}$ and $y_n = n$.

11. If f is uniformly continuous on $A \subseteq \mathbb{R}$ and $|f(x)| \geq k > 0$ for all $x \in A$, show that $\frac{1}{f}$ is uniformly continuous on A .

12. Suppose that the $\lim_{x \rightarrow c} f(x) = L$ where $L > 0$ and that $\lim_{x \rightarrow c} g(x) = \infty$. Show that $\lim_{x \rightarrow c} f(x)g(x) = \infty$.