

Exam 1 Math 341

Name _____

1. Prove that $n < 2^n$ for all $n \in \mathbb{N}$.

2. (a) What is the definition of an upper bound of S ?

(b) If u is an upper bound of S , under what condition is u a least upper bound?

3. If $S = \{2 - \frac{3}{n} : n \in \mathbb{N}\}$, prove that 2 is a least upper bound.

4. Prove that if (x_n) is a convergent sequence, then $\{x_n : n \in \mathbb{N}\}$ is bounded.

5. (a) Define the Nested Interval Property.

(b) State the Bolzano-Weierstrass Theorem.

(c) Give the definition of a Cauchy sequence.

6. If $\lim x_n = x$ and $\lim y_n = y$, prove that $\lim(x_n y_n) = xy$.

7. Suppose that (x_n) is a bounded increasing sequence. Prove that there is a number \tilde{x} such that $\lim x_n = \tilde{x}$.

8. (a) State the Squeeze Theorem.

(b) Show with all details how the Squeeze Theorem can be used to compute $\lim \frac{(-1)^n}{n^2}$.

9. (a) Use the fact that $\lim(1 + \frac{1}{n})^n = e$ to compute $\lim(1 + \frac{1}{n^2})^{3n^2}$.

(b) What theorem are you using to compute this limit?