Exam 1 Math 341 **Name** ________________

1. Prove that if (x_n) is a bounded increasing sequence, then there is a number x such that $\lim_{n\to\infty} x_n = x$.

2. Suppose that (y_n) is a sequence such that $\lim_{n\to\infty}(y_n) = y$ and $y \neq 0$. Show that there exists an integer K such that if $n > K$ then $|y_n| > |y|/2$.

3. Let $\lim_{n\to\infty}(y_n) = y$ where $y \neq 0$ and $y_n \neq 0$. Using the above result, show that $\lim_{n\to\infty}(1/y_n) = 1/y$.

4. Suppose (x_n) and (y_n) are sequences such that for some positive constant m, $x_n \geq m$. If $\lim_{n\to\infty} y_n = +\infty$, show that $\lim_{n\to\infty}(x_n\sqrt{y_n}) = \infty$.

5. Suppose that $0 < a < b$. Compute $\lim_{n\to\infty}\left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$.

6. Let $x_n = 1 + 1/2^2 + 1/3^2 + \cdots + 1/n^2$. Show that (x_n) is increasing and bounded and hence converges.

   Hint: If $k \geq 2$ then $1/k^2 \leq 1/k(k-1) = 1/(k-1) - 1/k$.

7. (a) State the Bolzano-Weierstrass Theorem.

(b) State the definition of a Cauchy sequence.