

1. Use the (ϵ, δ) -method to show that $\lim_{x \rightarrow 2} (x^2 + 3x) = 10$.

2. Determine if $\lim_{x \rightarrow 0^+} \sqrt{x} \cos\left(\frac{1}{x}\right)$ exists. Justify your answer.

3. Use the limit laws and composition laws to show that

$$\frac{\sqrt{2x + \sqrt{x}}}{3 + x}$$

is continuous on $[0, \infty)$.

4. State and prove the Boundedness Theorem.

5. (a) What does it mean to say that a function f on a set A is uniformly continuous?

(b) Show that $f(x) = x^2$ on \mathbf{R} is NOT uniformly continuous by using the sequences $x_n = n + \frac{1}{n}$ and $y_n = n$.

6. (a) What does it mean to say that a function $g(x)$ is Lipschitz on an interval I ?

(b) Show that $g(x) = \sqrt{x}$ is Lipschitz on $[1, \infty)$ by considering $|\sqrt{x} - \sqrt{y}|$.

7. (a) What does it mean to say that a function f has an absolute maximum?

(b) In the proof of the Maximum-Minimum Theorem, what is s^* .