Homework 1 Math 530 Due September 4, 2015

1. (a) Prove that $\left|\frac{z-w}{1-z\bar{w}}\right| < 1$ if $|z| < 1$ and $|w| < 1$.

   (b) Prove that $\left|\frac{z-w}{1-z\bar{w}}\right| = 1$ if either $|z| = 1$ or $|w| = 1$.

   What exception must be made if $|z| = |w| = 1$?

2. Suppose that $f(z) = u(x, y) + iv(x, y)$ is holomorphic at each z and that $f'(z) = 0$ for all z in $\mathbf{C}$. Use that

$$u_x + iv_x = v_y - iu_y = f'(z)$$

   to show that f is a constant.

3. Prove that an absolutely convergent series of complex numbers is convergent.

4. Use the formula that the radius of convergence R of a power series $\sum_{n=0}^{\infty} a_n z^n$ satisfies

$$R = \sup\{|r|; |a_n r^n| \text{ is a bounded sequence}\}$$

   to show that Hadamard's Formula gives the same value R.

5. (a) Find the power series (in powers of z) of $\sum_{n=0}^{\infty} \frac{4-3z}{3-2z}$

   (b) Find the power series of $\frac{1}{2-3z^2}$

   (c) Find the power series of $\frac{1}{(2-3z^2)^2}$

   (d) What is the radius of convergence for each series?