Homework 2 Math 530 Due September 21, 2015

1. Suppose that f is an entire function such that for some positive constant M and and a positive integer n, f satisfies

$$|f(z)| \leq M(1+|z|^n) \text{ for all } z \in \mathbf{C}.$$

Show that f is a polynomial.

2. Show that the function f defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

cannot be expressed as a power series in powers of z.

3. Compute the integral $\int_{|z|=1} e^z z^{-n}$ for all integers n.

4. Show that the successive derivatives of an analytic function at a point can never satisfy $|f^{(n)}(z)| > n!(n^n)$.

5. Let γ_1 denote the directed segment from 1 to $2+2i$, and let γ_2 be the semicircular path from $2+2i$ to $2i$ that lies in the upper half circle of radius 1 about $1+2i$. Compute $\int_{\gamma_1+\gamma_2} \frac{1}{z} dz$

6. Compute

$$\int_{|z|=2} \frac{dz}{z^2-1}$$