MA 174: Multivariable Calculus

EXAM I (practice)

NAME ______________________ INSTRUCTOR ______________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ___________

2. (5 pts) ___________

3. (5 pts) ___________

4. (5 pts) ___________

5. (5 pts) ___________

6. (5 pts) ___________

9. (5 pts) ___________

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9. (5 pts) ___________

Total Points: ___________

1. **Find the normal vector to the plane** $3x + 2y + 6z = 6$

   A. (0, 0, 1)

   B. [✓ (-3, -2, -6)]

   C. (1, 1, 1)

   D. (0, 0, 1)

   E. (1/3, 1/2, 1/6)

2. **Find distance from point** $(1, 1, 3)$ **to the plane** $3x + 2y + 6z = 6$

   A. [✓ 17/7]

   B. 3

   C. 12/5

   D. 0

   E. 11/7

3. **The surface defined by** $y^2 - x^2 = z$ **is a**

   A. [✓ hyperbolic paraboloid]

   B. elliptical cone

   C. elliptical paraboloid

   D. ellipsoid

   E. hyperboloid

4. Find the speed of the particle with position function $\vec{r}(t) = e^{3t}\,\mathbf{i} + e^{-3t}\,\mathbf{j} + te^{3t}\,\mathbf{k}$ when $t = 0$.

A. $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

B. 1

C. $\sqrt{2}$

D. $\sqrt{17}$

E. $\boxed{\sqrt{19}}$

5. The plane S passes through the point $P(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, and $z = 2 - t$. Which of the following vectors is normal to S?

A. $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

B. $\boxed{\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}}$

C. $\mathbf{i} + \mathbf{k}$

D. $\mathbf{i} - 2\,\mathbf{j}$

E. $\mathbf{i} + 2\,\mathbf{j}$

6. Which of the following statements is true for all three-dimensional vectors $\vec{a}, \vec{b}$, and $\vec{c}$, if θ is the angle between $\vec{a}$ and $\vec{b}$?

(i) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

(ii) $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{b} \times \vec{c}) \cdot \vec{a}$

(iii) $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot |\cos\theta|$

(iv) $(\vec{a} \times \vec{b}) \cdot a = 0$

A. All are true

B. (i) and (ii) only

C. (i), (ii), and (iv) only

D. (iii) and (iv) only

E. $\boxed{\textbf{(ii) and (iv) only}}$

7. The level surface of $f(x,y,z) = x^2 + y^2 - z^2$ corresponding to $f(x,y,z) \equiv 1$ intersects the xy-plane in a

A. $\boxed{\textbf{circle}}$

B. parabola

C. ellipse

D. hyperbola

E. line

8. If $L = \displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x+2y-3z}{\sqrt{x^2+y^2+z^2}}$**, then**

A. $L = 1$

B. $L = -2$

C. $L = -3$

D. $L = 0$

E. $\boxed{\textbf{the limit does not exist}}$

9. If $f(x,y) = \ln(x^2 + 2y^2)$**, then the partial derivative** f_{xy} **equals**

A. $\dfrac{4xy}{(x^2+2y^2)^2}$

B. $\dfrac{4(x^2-y^2)}{(x^2+2y^2)^2}$

C. $\boxed{\dfrac{-8xy}{(x^2+2y^2)^2}}$

D. $\dfrac{-4y}{(x^2+2y^2)^2}$

E. $\dfrac{-2x}{(x^2+2y^2)^2}$

10. A particle starts at the origin with initial velocity $\vec{i}+\vec{j}-\vec{k}$. Its acceleration is $\vec{a}(t)=t\vec{i}+\vec{j}+t\vec{k}$. Find its position at $t=1$.

A. $\frac{1}{6}\,\vec{i}+\frac{1}{2}\,\vec{j}+\frac{1}{3}\,\vec{k}$

B. $\frac{7}{6}\,\vec{i}+\frac{1}{2}\,\vec{j}-\frac{5}{6}\,\vec{k}$

C. $\vec{i}+\vec{j}+\vec{k}$

D. $\boxed{\frac{7}{6}\,\vec{i}+\frac{3}{2}\,\vec{j}-\frac{5}{6}\,\vec{k}}$

E. $\vec{i}+2\vec{j}-\vec{k}$

11. Find the arc length of the curve defined by $\vec{r}(t)=(t,\frac{\sqrt{6}}{2}t^2,t^3), -1\le t\le 1$.

A. 5

B. $\boxed{4}$

C. 3

D. 2

E. 6

12. Find the equation of the plane that contains the points $(1,2,1)$, $(2,-1,0)$ and $(3,3,1)$.

A. $-x-2y+9z=4$

B. $x+2y+7z=12$

C. $\boxed{x-2y+7z=4}$

D. $x+2y+z=6$

E. $-x+2y+9z=12$

13. Find parametric equations for the tangent line to the curve

$$\vec{r}(t) = (t^2 + 3t + 2, e^t \cos t, \ln(t+1))$$

at $t = 0$**.**

A. $\boxed{x = 2 + 3t \quad y = 1 + t \quad z = t}$

B. $x = 2t + 3,\ y = e^t(\cos t - \sin t),\ z = \dfrac{1}{t+1}$

C. $x = 3 + 2t \quad y = 1 + t \quad z = 1$

D. $x = 3t \quad y = 2t \quad z = 1 + t$

E. $x = 2 - t \quad y = 1 + t \quad z = 3 - 3t$

14. Let C **be the intersection of** $x^2 + y^2 = 16$ **and** $x + y + z = 5$**. Find the curvature at** $(0, 4, 1)$**.**

Answer: $\frac{1}{8}\sqrt{\frac{3}{2}}$

15. Find the equation for the surface consisting of all points P **for which the distance to the** x**–axis is twice the distance from** P **to the** yz**–plane. Identify the surface.**

Answer: $y^2 + z^2 = 4x^2$**, elliptical cone**

16. **Find an equation of the plane that passes through the point $P(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.**

    Answer: $x - 2y + 4z = -1$

17. **(a) Find the point of intersection of the lines $x = 2t + 1$, $y = 3t + 2$, $z = 4t + 3$ and $x = s + 2$, $y = 2s + 4$, $z = -4s - 1$.**

    (b) Find the plane determined by these lines.

    Answer: (a) $x = (1, 2, 3)$, **(b)** $20x - 12y - z = -7$

18. **A spring gun at ground level fires a golf ball at an angle of 45 degrees. The ball lands 10m away.**

    (a) What was the ball's initial speed?

    (b) For the same initial speed, find the two firing angles that make the range 6m.

    Recall that the Ideal Projectile Motion Equation is

$$\mathbf{r} = (v_0 cos\alpha)t\mathbf{i} + \left((v_0 sin\alpha)t - \frac{1}{2}gt^2\right)\mathbf{j}.$$

Answer: (a) $v_0 = \sqrt{10g}$,
Answer: (b) $\alpha = \frac{1}{2}\mathbf{arcsin}(\frac{3}{5}), \alpha = \pi - \frac{1}{2}\mathbf{arcsin}(\frac{3}{5})$

19. Find the unit tangent vector T, the principle unit normal vector N and the unit binormal vector B of $\mathbf{r(t)} = (\mathbf{3}\sin(\mathbf{t}))\mathbf{i} + (\mathbf{3}\cos(\mathbf{t})\mathbf{j} + \mathbf{4tk}$ at any t.

Recall: $\mathbf{N} = \frac{\frac{d\mathbf{T}}{dt}}{|\frac{d\mathbf{T}}{dt}|}$ and $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.

Answer: $\mathbf{T} = (\frac{\mathbf{3}}{\mathbf{5}}\cos(\mathbf{t}), -\frac{\mathbf{3}}{\mathbf{5}}\cos(\mathbf{t}), \frac{\mathbf{4}}{\mathbf{5}})$
Answer: $\mathbf{N} = (-\sin(\mathbf{t}), -\cos(\mathbf{t}), \mathbf{0})$
Answer: $\mathbf{B} = (\frac{\mathbf{4}}{\mathbf{5}}\cos(\mathbf{t}), -\frac{\mathbf{4}}{\mathbf{5}}\cos(\mathbf{t}), -\frac{\mathbf{3}}{\mathbf{5}})$

20. Calculate the tangential and normal components of the acceleration for $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + \frac{1}{3}t^3\vec{k}$.
Recall $a = a_T\mathbf{T} + a_N\mathbf{N}$ and $a_T = \frac{d}{dt}|v|$

Answer: $a_T = 2t, a_N = 2$

21. Find $\frac{\partial z}{\partial y}$ at $(1, \ln 2, \ln 3)$ if $z(x, y)$ is defined by the equation

$$xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$$

Answer: $-\frac{5}{3\ln 2}$