MA 174: Multivariable Calculus

EXAM II (practice)

NAME ____________________ INSTRUCTOR ____________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ________	**9. (5 pts)** ________
2. (5 pts) ________	**9. (5 pts)** ________
3. (5 pts) ________	**9. (5 pts)** ________
4. (5 pts) ________	**9. (5 pts)** ________
5. (5 pts) ________	**9. (5 pts)** ________
6. (5 pts) ________	**9. (5 pts)** ________

Total Points: ________

1. Suppose $z = f(x, y)$, where $x = e^t$ and $y = t^2 + 3t + 2$. Given that $\dfrac{\partial z}{\partial x} = 2xy^2 - y$ and $\dfrac{\partial z}{\partial y} = 2x^2y - x$, find $\dfrac{dz}{dt}$ when $t = 0$.

A. 3

B. 6

C. $\boxed{15}$

D. 9

E. -1

2. Find the directional derivative of the function $f(x, y, z) = x^2y^2z^6$ at the point $(1, 1, 1)$ in the direction of the vector $\langle 2, 1, -2 \rangle$.

A. -6

B. $\boxed{-2}$

C. 0

D. 2

E. 6

3. Find the direction in which the function $z = x^2 + 3xy - \frac{1}{2}\, y^2$ is increasing most rapidly at $(-1, -1)$.

A. $3i$

B. $5\vec{i} + 2\vec{j} - \vec{k}$

C. $\boxed{-5\vec{i} - 2\vec{j}}$

D. $2\vec{i} - 5\vec{j}$

E. $\sqrt{29}$

4. **If $xz^3 - xyz = 4$, find $\dfrac{\partial z}{\partial x}$.**

A. $\dfrac{\partial z}{\partial x} = \dfrac{xz}{z^3 - y^2}$

B. $\dfrac{\partial z}{\partial x} = \dfrac{3xz^2 - xy}{z^3 - yz}$

C. $\dfrac{\partial z}{\partial x} = 2x + xy$

D. $\boxed{\dfrac{\partial z}{\partial x} = \dfrac{yz - z^3}{3xz^2 - xy}}$

E. $\dfrac{\partial z}{\partial x} = z^3 - yz$

5. **The directional derivative of $f(x, y) = x^3 e^{-2y}$ in the direction of greatest increase of f at the point $(1, 0)$ is**

A. 6

B. 5

C. $\sqrt{5}$

D. 13

E. $\boxed{\sqrt{13}}$

6. **By using a linear approximation of $f(x, y) = \sqrt{x^2 + y}$ at $(4, 9)$, compute the approximate value of $f(5, 8)$.**

A. 5.2

B. 5.3

C. 5.5

D. $\boxed{5.7}$

E. 5.9

7. The max and min values of $f(x, y, z) = xyz$ on the surface $2x^2 + 2y^2 + z^2 = 2$ are

A. $\pm \frac{\sqrt{2}}{9}$

B. $\pm \frac{\sqrt{3}}{9}$

C. $\boxed{\pm \frac{\sqrt{6}}{9}}$

D. $\pm \frac{2\sqrt{2}}{9}$

E. $\pm \frac{2\sqrt{3}}{9}$

8. Find the maximum value of x^2+y^2 subject to the constraint $x^2-2x+y^2-4y=0$.

A. 0

B. 2

C. 4

D. 16

E. $\boxed{\mathbf{20}}$

9. If we use the method of Lagrange multipliers to find the maximum of $f(x, y) = 2x^2 - y^2 - y$ subject to the constraint $x^2 + y^2 = 1$, the Lagrange multipliers λ that we find are:

A. $\boxed{\lambda = 2}$

B. $\lambda = 0$

C. $\lambda = -1$

D. $\lambda = 2$ **and** $\lambda = -1$

E. $\lambda = 0$ **and** $\lambda = -1$

10. For the function $f(x, y) = x^3 + 2y^2 + xy - 2x + 5y$, the point $(-1, -1)$ yields

A. a local minimum

B. a local maximum

C. $\boxed{\text{a saddle point}}$

D. $\nabla f(-1, -1) \neq 0$

E. The Second Derivative Test gives no information at $(-1, -1)$

11. Use the method of reversing the order of integration to compute $\displaystyle\int_0^1 \int_{2x}^2 e^{y^2}\, dy dx$.

A. $\boxed{\frac{1}{4}(e^4 - 1)}$

B. $\frac{1}{2}(e^2 - 1)$

C. $\frac{1}{6}(e^3 - 1)$

D. $\frac{1}{2}(e^2 - e)$

E. $\frac{1}{4}(e^2 - e)$

12. A flat plate of constant density occupies the region in the xy–plane bounded by the curves $x = 0$ and $x = \sqrt{1 - y^2}$. If $(\overline{x}, \overline{y})$ is the center of mass, then $\overline{x}$ equals

A. $\frac{2}{3\pi}$

B. $\frac{1}{2}$

C. $\frac{2}{\pi}$

D. $\frac{3}{2\pi}$

E. $\boxed{\frac{4}{3\pi}}$

13. Find the volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

A. $\boxed{\frac{625}{12}}$

B. $\frac{625}{11}$

C. $\frac{542}{13}$

D. $\sqrt{15}\ \pi$

E. $\dfrac{\sqrt{8}\ \pi}{3}$

14. Which of the following integrals equals the volume of the solid bounded by $x = 0,\ y = 0,\ z = 0$ and $2x + y + z = 4$.

A. $\int_0^4 \int_0^4 \int_0^2 1 dx dy dz$

B. $\int_0^2 \int_0^{4-2x} \int_0^{4-y} 1 dz dy dx$

C. $\int_0^4 \int_0^{2x} \int_0^{4-y} 1 dz dy dx$

D. $\boxed{\int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} 1 dz dy dx}$

E. $\int_0^2 \int_0^1 \int_0^1 1 dz dx dy$

15. Evaluate $\iint_R (x + 2y) dA$ where R is the region of the plane bounded by $x + y = 2,\ x = y,\ y = 0$.

A. $1/3$

B. $\boxed{5/3}$

C. $7/3$

D. $11/3$

E. $14/3$

16. Let

$$S\colon x = u - v,\ \ y = uv,\ \ z = u + v^2$$

If $(0, b, 5)$ **is a point on the tangent plane to** S **at** $(0, 1, 2)$ **on** S**, then** $b =$

A. $\boxed{3}$

B. 1

C. -2

D. 0

E. 2

17. Find $\left(\dfrac{\partial w}{\partial y}\right)_x$ **at** $(w, x, y, z) = (4, 2, 1, -1)$ **if**

$$w = x^2y^2 + yz - z^3, \quad x^2 + y^2 + z^2 = 6$$

A. -1

B. 1

C. 3

D. $\boxed{5}$

E. 7

18. Consider the function $f(x, y) = 2x^2 - 3xy + y^2$**. Find two unit vectors such that the directional derivative of** f **at the point** $(1, 1)$ **in these two directions is 1.**

Answer: $(1, 0)$ **and** $(0, -1)$

19. Find cubic approximation of $f(x,y) = \dfrac{1}{1-x-y+xy}$ **near the origin.**

Answer: $1 + x + y + x^2 + xy + y^2 + x^3 + x^2y + xy^2 + y^3$

20. Find a equation for the tangent plane of

$$\cos(\pi x) - x^2y + e^{xz} + yz = 4 \quad \textbf{at} \quad (0,1,2)$$

Answer: $2x + 2y + z - 4 = 0$

21. Find absolute maximum and minimum values of

$$f(x,y) = x^2 + 2y^2 - x$$

on the disc $x^2 + y^2 \leq 1$**.**

Answer: max=$\frac{9}{4}$**, min=**$-\frac{1}{4}$