MA 174: Multivariable Calculus

Final EXAM (practice)

NAME ________________________ Class Meeting Time: ________________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) __________	**9. (5 pts)** __________
2. (5 pts) __________	**9. (5 pts)** __________
3. (5 pts) __________	**9. (5 pts)** __________
4. (5 pts) __________	**9. (5 pts)** __________
5. (5 pts) __________	**9. (5 pts)** __________
6. (5 pts) __________	**9. (5 pts)** __________

Total Points: __________

Surface Integral:

If R is the shadow region of a surface S defined by the equation $f(x, y, z) = c$, and g is a continuous function defined at the points of S, then the integral of g over S is the integral

$$\int\int_S g(x, y, z)\, d\sigma = \int\int_R g(x, y, z) \frac{|\nabla f|}{|\nabla f \cdot \mathbf{p}|} dA,$$

where $\mathbf{p}$ is a unit vector normal to R and $|\nabla f \cdot \mathbf{p}| \neq 0$.

Green's Theorem:

$$\oint_C P\, dx + Q\, dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

where C is a positively oriented simple closed curve enclosing region R, and P, Q have continuous partial derivatives.

Divergence Theorem:

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma = \iiint_D \nabla \cdot \mathbf{F}\, dV$$

where D is a simple solid region with boundary S given outward orientation, and component functions of $\mathbf{F}$ have continuous partial derivatives.

Stokes' Theorem:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\ d\sigma$$

where C, given counterclockwise direction, is the boundary of oriented surface S, $\mathbf{n}$ is the surface's unit normal vector and component functions of $\mathbf{F}$ have continuous partial derivatives.

1. The arclength of the curve $\vec{r}(t) = \frac{2}{3}\, t^{3/2}\vec{i} + \frac{2}{3}(2-t)^{3/2}\vec{j} + (t-1)\vec{k}$ for $\frac{1}{4} \le t \le \frac{1}{2}$ is:

A. $\sqrt{2}/4$

B. $\boxed{\sqrt{3}/4}$

C. $\sqrt{2}/2$

D. $3/2$

E. $1/2$

2. Find the directional derivative of the function $f(x, y, z) = x^2y^2z^6$ at the point $(1, 1, 1)$ in the direction of the vector $\langle 2, 1, -2\rangle$.

A. -6

B. $\boxed{-2}$

C. 0

D. 2

E. 6

3. The function $f(x, y) = 3x + 12y - x^3 - y^3$ has

A. no critical point

B. exactly one saddle point

C. $\boxed{\textbf{two saddle points}}$

D. two local minimum points

E. two local maximum points

4. The function $f(x, y) = x^3 + y^3 - 3xy$ has how many critical points?

A. None

B. One

C. $\boxed{\textbf{Two}}$

D. Three

E. More than three

5. The max and min values of $f(x,y,z) = xyz$ on the surface $2x^2 + 2y^2 + z^2 = 2$ are

A. $\pm \dfrac{\sqrt{2}}{9}$

B. $\pm \dfrac{\sqrt{3}}{9}$

C. $\boxed{\pm \dfrac{\sqrt{6}}{9}}$

D. $\pm \dfrac{2\sqrt{2}}{9}$

E. $\pm \dfrac{2\sqrt{3}}{9}$

6. Find the maximum value of x^2+y^2 subject to the constraint $x^2-2x+y^2-4y=0$.

A. **0**

B. **2**

C. **4**

D. **16**

E. $\boxed{\textbf{20}}$

7. Find the parametric equations for the line passing through $P = (2,1,-1)$, and normal to the tangent plane of

$$4x + y^2 + z^3 = 8$$

at P.

A. $x = t+4, y = t, z = -t$

B. $\boxed{x = 4t+2, y = 2t+1, z = 3t-1}$

C. $\dfrac{x-2}{4} = \dfrac{y-1}{2} = \dfrac{z-1}{3}$

D. $\dfrac{x-4}{2} = \dfrac{y-3}{9} = \dfrac{2-3}{-1}$

E. $x = 4t-2,\ \ y = 2t-1,\ \ z = -3t+1$

8. One vector perpendicular to the plane that is tangent to the surface $2x^2 + xy^2 + z^3 = 2$ at the point $(-1,1,1)$ is:

A. $\boxed{-3\vec{i} - 2\vec{j} + 3\vec{k}}$

B. $-\vec{+}\vec{j} + \vec{k}$

C. $-\vec{+}5\vec{k}$

D. $2\vec{-}\vec{j} + \vec{k}$

E. $5\vec{i} + 2\vec{j} + 3\vec{k}$

9. Suppose $z = f(x, y)$, where $x = e^t$ and $y = t^2 + 3t + 2$. Given that $\dfrac{\partial z}{\partial x} = 2xy^2 - y$ and $\dfrac{\partial z}{\partial y} = 2x^2y - x$, find $\dfrac{dz}{dt}$ when $t = 0$.

A. **3**

B. **6**

C. $\boxed{\textbf{15}}$

D. **9**

E. -1

10. Find the equation in spherical coordinates for $x^2 + y^2 = x$.

A. $\rho = \sin\phi\cos\theta$

B. $\rho\sin\phi = \sin^2\phi\cos\theta$

C. $\rho = \sin\phi\cos\phi$

D. $\rho^2 = \rho\cos\phi$

E. $\boxed{\rho^2\sin^2\phi = \rho\sin\phi\cos\theta}$

11. Let $S\colon x = u - v,\ y = uv,\ z = u + v^2$. If $(0, b, 5)$ is a point on the tangent plane to S at $(0, 1, 2)$ on S, then $b =$

A. $\boxed{3}$

B. 1

C. -2

D. 0

E. 2

12. Find the area of the region bounded by $x = y - y^2$ and $x + y = 0$

A. $1/3$

B. $2/.3$

C. 1

D. $\boxed{4/3}$

E. $5/3$

13. Find the area in the plane that lies inside the curve $r = 1 + \cos\theta$ and outside the circle $r = 1$.

A. $\pi/2$

B. $1 + \pi/2$

C. $1 + \pi/4$

D. $2 + \pi/2$

E. $\boxed{2 + \pi/4}$

14. A sheet of metal occupies the region bounded by the x–axis and the parabola $y = 1 - x^2$. At each point, the density is equal to the distance from the y–axis. Find the mass of the sheet.

A. $1/4$

B. $1/3$

C. $\boxed{1/2}$

D. $2/3$

E. 1

15. Evaluate $\int_C ydx + xdy + 2zdz$, where

$$C\colon F(t) = t(t-1)e^{\sqrt{t}}\ \vec{i} + \sin(\frac{\pi}{2}\ t^2)\vec{j} + \frac{t}{t^2+1}\ \vec{k}, \quad 0 \le t \le 1.$$

A. 1

B. $\frac{1}{2}$

C. $\boxed{\frac{1}{4}}$

D. 0

E. -1

16. Let C be the boundary of the triangle with vertices $(0,0),\ (1,0),\ (1,1)$ oriented counterclockwise. Then $\int_C ydx - xdy =$

A. $\boxed{-1}$

B. 0

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

E. 2

17. **Let** $\vec{F} = \nabla f,\ f = \sqrt{x^2+y^2}$**. If** C **is any smooth curve joining the points** $(1,1),\ (2,2)$**, then** $\int_C 2\vec{F}\cdot d\vec{r} =$

A. $\sqrt{2}$
B. $\sqrt{12}$
C. $-\sqrt{2}$
D. 1
E. $\boxed{2\sqrt{2}}$

18. **Let** D **be the solid region bounded by the surfaces** $x^2+z^2=4,\ y=1,\ y=0$**, and** S **be the boundary of** D**. If** $\vec{F}(x,y,z) = \frac{1}{3}\,(x^3\ \vec{i} + y^3\vec{j} + z^3\ \vec{k})$**, then with** $\vec{n}$ **being the unit outward normal, evaluate** $\iint_S \vec{F}\cdot\vec{n}d\sigma$**.**

A. 8π
B. $\boxed{\frac{28}{3}\ \pi}$
C. 28π
D. 10π
E. 20

19. **Find** a, b **in the following formula which connect the triple integral from rectangular coordinates to spherical coordinate**

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} ydzdydx = \int_0^{\pi/2}\int_a^{\pi/2}\int_0^{3\csc\varphi} bd\rho d\varphi d\theta.$$

A. $a=0,\ b=\rho^2\sin\varphi$
B. $a=\pi/4,\ b=\rho^3\sin\varphi\sin\theta$
C. $\boxed{a=\pi/4,\ b=\rho^3\sin^2\varphi\sin\theta}$
D. $a=\frac{\pi}{3},\ b=\rho^3\sin^2\varphi\sin\theta$
E. $a=-\pi/2,\ b=\rho^3\sin^2\varphi$

20. $\vec{F} = 2xy\vec{i} + (x^2+3y^2)\vec{j}$ **is a conservative vector field, i.e.,** $\vec{F} = \nabla f$**. If** $f(0,0)=0$**, then** $f(1,1) =$

A. 1
B. $\boxed{2}$
C. 3
D. 2
E. 4

21. Evaluate $\iint_S y\,dS$**, where** S **is the part of the plane** $x+2y+z=1$ **in the 1st octant.**

A. $\dfrac{1}{2\sqrt{6}}$

B. $\dfrac{1}{2}$

C. $\boxed{\dfrac{\sqrt{6}}{24}}$

D. $\sqrt{5}$

E. $\dfrac{\sqrt{5}}{24}$

22. If $\vec{F}(x,y,z) = xz\vec{i} + xyz\ \vec{j} - y^2\vec{k}$**, then curl** $\vec{F}$ **evaluated at** $(1,1,1)$ **equals**

A. $3\vec{i} - \vec{j} + \vec{k}$

B. $3\vec{i} + \vec{j} - \vec{k}$

C. $\vec{i} + \vec{j} - \vec{k}$

D. $\boxed{-3\vec{i} + \vec{j} + \vec{k}}$

E. $\vec{i} - \vec{j} + 2\vec{k}$

23. Evaluate $\int_0^2 \int_x^2 e^{y^2}\,dy\,dx$**.**

A. $2(e^4 - 1)$

B. $e^4 - 1$

C. $\dfrac{e^4}{2}$

D. $\boxed{\dfrac{e^4 - 1}{2}}$

E. $e^4 + 1$

24. Let R be the region in the xy–plane bounded by $y = x,\ y = -x$ and $y = \sqrt{4 - x^2}$. Evaluate the integral

$$\iint_R y dA.$$

A. $\dfrac{8\sqrt{3}}{2}$

B. $\dfrac{8}{3\sqrt{2}}$

C. $\dfrac{4}{\sqrt{2}}$

D. $\boxed{\dfrac{8\sqrt{2}}{3}}$

E. $4\sqrt{2}$

25. Find the surface area of the part of the surface $z = x^2 + y^2$ below the plane $z = 9$.

A. $\frac{\pi}{4}(3\sqrt{3} - 1)$

B. $\frac{\pi}{4}(3\sqrt{3} - 2\sqrt{2})$

C. $\boxed{\frac{\pi}{6}(37^{3/2} - 1)}$

D. $\frac{\pi}{6}(29^{3/2} - 1)$

E. $\frac{\pi}{6}(2y^{3/2} - 1)$

26. Find a, b such that

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^2 z^2 x dz dy dx = \int_0^2 \int_0^a \int_0^b z^2 x \ dx dy dz.$$

A. $a = 3,\ b = x$

B. $a = \sqrt{9 - z^2},\ b = 3$

C. $\boxed{a = 3,\ b = \sqrt{9 - y^2}}$

D. $a = z,\ b = 3$

E. $a = 3,\ b = \sqrt{9 - x^2}$

27. If $\vec{F}(x,y,z) = (x\sin x + y)\vec{i} + xy\vec{j} + (yz+x)\vec{k}$, then curl $\vec{F}$ evaluated at $(\pi, 0, 2)$ equals

A. $\pi\vec{i} - \vec{j} + \vec{k}$

B. $\boxed{2\vec{i} - \vec{j} - \vec{k}}$

C. $2\vec{i} - \pi\vec{j} + \vec{k}$

D. $2\vec{i} - \vec{j} + \pi\vec{k}$

E. $2\vec{i} + \vec{j} + \vec{k}$

28. Evaluate $\int_C (2x+yz)dx + (2y+xz)dy + xydz$

where c: $\vec{r}(t) = t^2(1+t)\vec{i} + \cos\left(\frac{\pi}{2}t^2\right)\vec{j} + \frac{t^2+1}{t^4+1}\ \vec{k}$, $0 \le t \le 1$.

A. 1

B. 2

C. $\boxed{\mathbf{3}}$

D. 4

E. 5

29. Evaluate $\iint_S (x^2+y^2+z^2)dS$ **where** S **is the upper hemisphere of** $x^2+y^2+z^2=2$.

A. 12π

B. $\boxed{8\pi}$

C. 6π

D. 4π

E. 3π

30. **Evaluate** $\int_C -\frac{2y}{x^2+y^2}\,dx + \frac{2x}{x^2+y^2}\,dy$ **where** C **is the circle** $x^2+y^2=1$ **oriented counterclockwise.**

A. 2π

B. $\boxed{4\pi\text{, No to Green's theorem because the function is not continues at origin}}$

C. 0

D. -4π

E. -2π

31. **Calculate the surface integral** $\iint_S \vec{F}\cdot\vec{n}\,dS$ **where** S **is the sphere** $x^2+y^2+z^2=2$ **oriented by the outward normal and** $\vec{F}(x,y,z) = 5x^3\vec{i} + 5y^3\vec{j} + 5z^3\vec{k}$**.**

A. $\boxed{48\sqrt{2}\pi}$

B. 16π

C. 24π

D. $25\sqrt{2}\pi$

E. 20π

32. **What is the spherical coordinates** $(\rho,\varphi,\theta) =$ ____________________ **and the cylindircal coordinates** $(r,\theta,z) =$ ____________________ **for the point** $(x,y,z) = (1,1,1)$**?**

Answer: $(\rho,\varphi,\theta) = (\sqrt{3}, \cos^{-1}(\frac{1}{\sqrt{3}}), \frac{\pi}{4})$

Answer: $(r,\theta,z) = (\sqrt{2}, \frac{\pi}{4}, 1)$