**MA 265: Linear Algebra**

**EXAM I (Practice)**

**Feb. 14, 2008**

NAME ______________________ Lecture Time ______________________

**NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.**

**Points awarded**

**1. (6 pts)** ___________

**2. (6 pts)** ___________

**3. (6 pts)** ___________

**4. (6 pts)** ___________

**5. (6 pts)** ___________

**6. (6 pts)** ___________

**7. (6 pts)** ___________

**8. (6 pts)** ___________

**9. (6 pts)** ___________

**10. (6 pts)** ___________

**11. (6 pts)** ___________

**12. (6 pts)** ___________

**13. (6 pts)** ___________

**14. (6 pts)** ___________

**15. (6 pts)** ___________

**16. (6 pts)** ___________

**17. (6 pts)** ___________

**Total Points:** ___________/102

**1. If** $\begin{bmatrix} a-b & a+b \\ c-d & c+d \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 1 & -7 \end{bmatrix}$**, find** $a, b, c$**, and** $d$**.**

**A.** $\boxed{a = 7, \quad b = 3, \quad c = -3, \quad d = -4.}$

**B.** $a = -3, \quad b = 3, \quad c = 4, \quad d = 7.$

**C.** $a = -7, \quad b = -3, \quad c = -3, \quad d = -4.$

**D.** $a = 10, \quad b = 6, \quad c = -2, \quad d = -5.$

**E.** $a = 10, \quad b = 6, \quad c = 9, \quad d = 8.$

**2. Let** $A = [-1 \;\; 2 \;\; 4], \; B = [1 \;\; 2 \;\; -3]$ **and** $C = [-3 \;\; 0 \;\; 1]$**. Compute** $(BA^T)C$

**A.** $[-20 \;\; 0 \;\; -8]$

**B.** $[20 \;\; 0 \;\; 8]$

**C.** $[-27 \;\; 0 \;\; 9]$

**D.** $\boxed{[27 \;\; 0 \;\; -9]}$

**E.** $[11 \;\; 22 \;\; -33]$

**3. For what values of** $a, b$**, and** $c$ **is the following matrix symmetric:**

$$\begin{bmatrix} 1 & 1 & a+1 \\ 1 & c & 2b \\ 2 & 4 & 1 \end{bmatrix}$$

**A.** $a = \frac{1}{2}, \quad b = 2, \quad c = 1$

**B.** $\boxed{a = 1, \quad b = 2, \quad c = \textbf{any number}}$

**C.** $a = \frac{1}{2}, \quad b = 1, \quad c =$ **any number**

**D.** $a = \frac{1}{2}, \quad b = 1, \quad c = 1$

**E. There is no such** $a, b$**, and** $c$

**4. Given** $A = \begin{bmatrix} 1+i & 2-i \\ -3 & 1-2i \end{bmatrix}$ **and** $B = \begin{bmatrix} 2i & 1-i \\ 2+3i & 2 \end{bmatrix}$**, then** $AB =$

**A.** $\begin{bmatrix} 5-6i & 6-2i \\ 8-7i & -1+i \end{bmatrix}$

**B.** $\boxed{\begin{bmatrix} 5+6i & 6-2i \\ 8-7i & -1-i \end{bmatrix}}$

**C.** $\begin{bmatrix} 5-6i & 6+2i \\ 8-7i & -1+i \end{bmatrix}$

**D.** $\begin{bmatrix} 5+6i & 6-2i \\ 8-7i & 1-i \end{bmatrix}$

**E.** $\begin{bmatrix} 5-6i & -6+2i \\ 8-7i & -1+i \end{bmatrix}$

**5. The reduced row echelon form of** $A = \begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 1 & 2 & 1 & -3 & 2 \\ 1 & 2 & 0 & -3 & 1 \\ 3 & 6 & 1 & -9 & 3 \end{bmatrix}$ **is**

**A.** $\begin{bmatrix} 1 & 0 & 0 & -3 & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

**B.** $\begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

**C.** $\boxed{\begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$

**D.** $\begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

**E.** $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

**6. Let**

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \textbf{and} \quad \mathbf{x}_i = A^{-1}\mathbf{b}_i$$

**with** $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **and** $\mathbf{b}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. **Which of the following is true?**

**A.** $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

**B.** $\mathbf{x}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$

**C.** $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

**D.** $\boxed{\mathbf{x}_1 = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}$

**E.** $\mathbf{x}_1 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

**7. Suppose that the system of equations**

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + z &= b \\ 5x + 9y - 8z &= c \end{aligned}$$

**has a solution. Which equation must** $a, b$, **and** $c$ **satisfy:**

**A.** $3a - b + c = 0$

**B.** $a - 3b + c = 0$

**C.** $\boxed{3a + b - c = 0}$

**D.** $7a - b - c = 0$

**E.** $a + b + c = 0$

8. **What value(s) of $a$ can make the following linear equation system INCONSISTENT.**

$$\begin{cases} x + y - z = 2 \\ -x - 2y + z = 3 \\ x + y + (a^2 - 5)z = a \end{cases}$$

**A.** $\boxed{a = 2.}$

**B.** $a = -2$.

**C.** $a = 2$ **or** $a = -2$.

**D.** $a \neq 2$ **and** $a \neq 2$.

**E. None of the above.**

9. **For what value(s) of $t$ does the system with the following augmented matrix have no solution:**

$$\begin{bmatrix} 0 & t+3 & 0 & 6 \\ 0 & 2 & 2t-2 & -2 \\ 1 & -1 & -2 & 1 \end{bmatrix}.$$

**A.** $t = 1$ **and** $t = -3$

**B.** $\boxed{t = 1 \text{ and } t = 3}$

**C.** $t = 9$

**D.** $t = -1$ **and** $t = 3$

**E.** $t = -9$

10. **(True or False) The following statements is always TRUE.**

**A. If A is a singular matrix, then $Ax = 0$ has only trivial solution.**

**B. If A is non-invertible, then $Ax = 0$ has only trivial solution.**

**C. If A is invertible, then $A^{-1}x = 0$ does not have trivial solution.**

**D. If A is a non-singular matrix, then $A^{-1}x = 0$ has only trivial solution.**

**E. If A is a non-singular matrix, then $A^{-1}x = 0$ has non-trivial solutions.**

**Answer: F, F, F, T, F**

**11. If** $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & a & 1 \\ 0 & 0 & 1 \end{bmatrix}$**, then** $A^{-1}$ **has the form** $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$**.**
**Find the sum** $a + b + c$**.**

**A.** $0$

**B.** $1$

**C.** $\boxed{-1}$

**D.** $2$

**E.** $-2$

**12. Given** $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$**, then** $A^{-1} =$

**A.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$

**B.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\phi & \sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$

**C.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$

**D.** $\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}}$

**E.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi & -\cos\phi \end{bmatrix}$

**13. For which value(s) of $a$ is the matrix $A = \begin{bmatrix} 0 & 1 & a \\ 1 & 3 & 5 \\ a & 2 & a \end{bmatrix}$ singular?**

- **A.** $a = -1$**,** $a = 0$
- **B.** $a = 0$ **and** $a = -2$
- **C.** $\boxed{a = 0 \textbf{ and } a = 2}$
- **D.** $a = 2$
- **E.** $a = 0$ **and** $a = 1$

**14. Let $A$ and $B$ be $3\times3$ matrices with $\det A = 3$ and $\det B = 2$. Then $\det(A\ \textbf{adj}\ BA^TB^{-1})$ is**

- **A.** 9
- **B.** $\boxed{18}$
- **C.** 36
- **D.** 1/2
- **E.** 1

**15. Let $A$ be a $n \times n$ invertible matrix. Which of the following statements are NOT true:**

**(i).** $A^T$ **is nonsingular.**

**(ii).** $A\mathrm{x} = \mathbf{0}$ **has only trivial solution.**

**(iii).** $A$ **is row equivalent to a matrix** $B$ **that has a row of zeros.**

**(iv).** $A\mathrm{x} = \mathrm{b}$ **has a unique solution for every** $n \times 1$ **matrix b.**

**(v).** $|A| \neq 0$**.**

**A. (i) and (iii).**

**B. (ii) and (v).**

**C.** $\boxed{\textbf{(iii) only}}$**.**

**D. (iii) and (iv).**

**E. (v) only.**

**16. Given** $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 3$**, then** $\begin{vmatrix} -x & -y & -z \\ 3p+a & 3q+b & 3r+c \\ 2p & 2q & 2r \end{vmatrix} =$

**A.** $\boxed{-6}$

**B.** 6

**C.** 18

**D.** $-18$

**E.** $-3$

**17. Find adj**$(A)$ **if** $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & 3 \\ 2 & 1 & 5 \end{bmatrix}$

**A.** $\boxed{\begin{bmatrix} -8 & -6 & 10 \\ 6 & -3 & -3 \\ 2 & 3 & -1 \end{bmatrix}}$

**B.** $\begin{bmatrix} -8 & 6 & 2 \\ -6 & -3 & -3 \\ 10 & -3 & -1 \end{bmatrix}$

**C.** $\begin{bmatrix} -8 & 6 & 2 \\ -6 & 3 & 3 \\ 10 & -3 & -1 \end{bmatrix}$

**D.** $\begin{bmatrix} -8 & 6 & 10 \\ 6 & -3 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

**E.** $\begin{bmatrix} -8 & 6 & 2 \\ -6 & -3 & 3 \\ -10 & -3 & -1 \end{bmatrix}$

**18. For** $n \times n$ **matrices with real entires** $A$ **and** $B$**, decide which statements are ALWAYS true.**

**I.** $\det(A + B) = \det(A) + \det(B)$

**II. If** $\det(A) \neq 0$ **then** $\det(adj(A)) = \dfrac{1}{\det(A)}$

**III. For any real number** $c$**,** $\det(cA) = c\det(A)$**.**

**A.** $\boxed{\textbf{None of the statements are true}}$

**B. I only**

**C. II only**

**D. III only**

**E. II and III only**

**19. The matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is nonsingular if and only if**

**A.** $a \neq 0,\ b \neq 0$

**B.** $\boxed{ad - bc \neq 0}$

**C.** $a = b = c = d = 1$

**D.** $ad - bc = 0$

**E.** $a = b = c = d$

**20. If $A$ is an $n \times n$ singular matrix, then $A(\textbf{adj}A) =$**
**(Note that $O_{n\times n}$ is the $n \times n$ matrix with all zeros while 0 means just the number 0).**

**A.** $\boxed{O_{n\times n}}$

**B.** $I_n$

**C.** $0$

**D.** $A^{-1}$

**E.** $1$

**21. Consider the solution $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to the system $\begin{bmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Decide which equality below is true for all $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ in $\mathbf{R}^3$.**

**A.** $x_2 = \begin{vmatrix} 11 & a & 13 \\ 4 & b & 8 \\ 3 & c & 9 \end{vmatrix}$

**B.** $x_1 = \begin{vmatrix} a & b & c \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}$

**C.** $\boxed{x_3 = -\frac{\begin{vmatrix} 4 & 12 & b \\ 11 & 7 & a \\ 3 & 7 & c \end{vmatrix}}{\begin{vmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}}}$

**D.** $x_2 = \frac{b}{\begin{vmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}}$

**E.** $x_1 = \frac{\begin{vmatrix} a & 4 & 3 \\ b & 12 & 7 \\ c & 8 & 9 \end{vmatrix}}{\begin{vmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}}$

**22. The heads of three vectors OA, OB and OC (where O is the origin) are: $A = (1, 2, 5)$, $B = (5, 0, -1)$ and $C = (-2, 2, 1)$ and they form a triangle, which side(s) of the triangle ABC is (are) the longest.**

**A.** AB

**B.** AC

**C.** $\boxed{\text{BC}}$

**D.** AB and AC

**E.** AB and BC

**23. Which of the following subset of $\mathrm{R}^2$ with the usual operations of vector addition and scalar multiplication are subspaces?**

- **I.** $W_1$ **is the set of all vectors of the form** $\begin{bmatrix} x \\ y \end{bmatrix}$**, where** $x \geq 0$.
- **II.** $W_2$ **is the set of all vectors of the form** $\begin{bmatrix} x \\ y \end{bmatrix}$**, where** $x \geq 0$ **and** $y \geq 0$.
- **III.** $W_3$ **is the set of all vectors of the form** $\begin{bmatrix} x \\ y \end{bmatrix}$**, where** $x = 0$.

- **A.** $W_1$ **only;**
- **B.** $W_2$ **only;**
- **C.** $\boxed{W_3 \textbf{ only;}}$
- **D.** $W_1$ **and** $W_2$**;**
- **E.** $W_2$ **and** $W_3$**.**

**24. Which of the following subsets of the vector space $M_{nn}$ are subspace?**

- **I. The set of all** $n \times n$ **symmetric matrices;**
- **II. The set of all** $n \times n$ **diagonal matrices;**
- **III. The set of all** $n \times n$ **singular matrices;**

- **A.** $I$ **only;**
- **B.** $II$ **only;**
- **C.** $III$ **only;**
- **D.** $\boxed{I \textbf{ and } II}$**;**
- **E.** $II$ **and** $III$**.**