MA 265: Linear Algebra

EXAM I (Practice)

Feb. 14, 2008

NAME ____________________ Lecture Time ____________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (6 pts) ________	**10. (6 pts)** ________
2. (6 pts) ________	**11. (6 pts)** ________
3. (6 pts) ________	**12. (6 pts)** ________
4. (6 pts) ________	**13. (6 pts)** ________
5. (6 pts) ________	**14. (6 pts)** ________
6. (6 pts) ________	**15. (6 pts)** ________
7. (6 pts) ________	**16. (6 pts)** ________
8. (6 pts) ________	**17. (6 pts)** ________
9. (6 pts) ________	

Total Points: ________ /102

1. **If** $\begin{bmatrix} a-b & a+b \\ c-d & c+d \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 1 & -7 \end{bmatrix}$**, find** a, b, c**, and** d**.**

A. $a = 7, \quad b = 3, \quad c = -3, \quad d = -4$**.**

B. $a = -3, \quad b = 3, \quad c = 4, \quad d = 7$**.**

C. $a = -7, \quad b = -3, \quad c = -3, \quad d = -4$**.**

D. $a = 10, \quad b = 6, \quad c = -2, \quad d = -5$**.**

E. $a = 10, \quad b = 6, \quad c = 9, \quad d = 8$**.**

2. **Let** $A = [-1 \;\; 2 \;\; 4], \; B = [1 \;\; 2 \;\; -3]$ **and** $C = [-3 \;\; 0 \;\; 1]$**. Compute** $(BA^T)C$

A. $[-20 \;\; 0 \;\; -8]$

B. $[20 \;\; 0 \;\; 8]$

C. $[-27 \;\; 0 \;\; 9]$

D. $[27 \;\; 0 \;\; -9]$

E. $[11 \;\; 22 \;\; -33]$

3. **For what values of** a, b**, and** c **is the following matrix symmetric:**

$$\begin{bmatrix} 1 & 1 & a+1 \\ 1 & c & 2b \\ 2 & 4 & 1 \end{bmatrix}$$

A. $a = \frac{1}{2}, \quad b = 2, \quad c = 1$

B. $a = 1, \quad b = 2, \quad c =$ **any number**

C. $a = \frac{1}{2}, \quad b = 1, \quad c =$ **any number**

D. $a = \frac{1}{2}, \quad b = 1, \quad c = 1$

E. **There is no such** $a, b,$ **and** c

4. Given $A = \begin{bmatrix} 1+i & 2-i \\ -3 & 1-2i \end{bmatrix}$ **and** $B = \begin{bmatrix} 2i & 1-i \\ 2+3i & 2 \end{bmatrix}$, **then** $AB =$

A. $\begin{bmatrix} 5-6i & 6-2i \\ 8-7i & -1+i \end{bmatrix}$

B. $\begin{bmatrix} 5+6i & 6-2i \\ 8-7i & -1-i \end{bmatrix}$

C. $\begin{bmatrix} 5-6i & 6+2i \\ 8-7i & -1+i \end{bmatrix}$

D. $\begin{bmatrix} 5+6i & 6-2i \\ 8-7i & 1-i \end{bmatrix}$

E. $\begin{bmatrix} 5-6i & -6+2i \\ 8-7i & -1+i \end{bmatrix}$

5. The reduced row echelon form of $A = \begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 1 & 2 & 1 & -3 & 2 \\ 1 & 2 & 0 & -3 & 1 \\ 3 & 6 & 1 & -9 & 3 \end{bmatrix}$ **is**

A. $\begin{bmatrix} 1 & 0 & 0 & -3 & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

6. Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \textbf{and} \quad \mathbf{x}_i = A^{-1}\mathbf{b}_i$$

with $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **and** $\mathbf{b}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. **Which of the following is true?**

A. $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

B. $\mathbf{x}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$

C. $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

D. $\mathbf{x}_1 = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

E. $\mathbf{x}_1 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

7. Suppose that the system of equations

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + z &= b \\ 5x + 9y - 8z &= c \end{aligned}$$

has a solution. Which equation must a, b**, and** c **satisfy:**

A. $3a - b + c = 0$

B. $a - 3b + c = 0$

C. $3a + b - c = 0$

D. $7a - b - c = 0$

E. $a + b + c = 0$

8. What value(s) of a can make the following linear equation system INCONSISTENT.

$$\begin{cases} x + y - z = 2 \\ -x - 2y + z = 3 \\ x + y + (a^2 - 5)z = a \end{cases}$$

A. $a = 2$.

B. $a = -2$.

C. $a = 2$ **or** $a = -2$.

D. $a \neq 2$ **and** $a \neq 2$.

E. None of the above.

9. For what value(s) of t does the system with the following augmented matrix have no solution:

$$\begin{bmatrix} 0 & t+3 & 0 & 6 \\ 0 & 2 & 2t-2 & -2 \\ 1 & -1 & -2 & 1 \end{bmatrix}.$$

A. $t = 1$ **and** $t = -3$

B. $t = 1$ **and** $t = 3$

C. $t = 9$

D. $t = -1$ **and** $t = 3$

E. $t = -9$

10. (True or False) The following statements is always TRUE.

A. If A is a singular matrix, then $Ax = 0$ has only trivial solution.

B. If A is non-invertible, then $Ax = 0$ has only trivial solution.

C. If A is invertible, then $A^{-1}x = 0$ does not have trivial solution.

D. If A is a non-singular matrix, then $A^{-1}x = 0$ has only trivial solution.

E. If A is a non-singular matrix, then $A^{-1}x = 0$ has non-trivial solutions.

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & a & 1 \\ 0 & 0 & 1 \end{bmatrix}$**, then** A^{-1} **has the form** $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$**.**
Find the sum $a + b + c$**.**

A. 0

B. 1

C. -1

D. 2

E. -2

12. Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$**, then** $A^{-1} =$

A. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\phi & \sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi & -\cos\phi \end{bmatrix}$

13. For which value(s) of a is the matrix $A = \begin{bmatrix} 0 & 1 & a \\ 1 & 3 & 5 \\ a & 2 & a \end{bmatrix}$ singular?

A. $a = -1$**,** $a = 0$

B. $a = 0$ **and** $a = -2$

C. $a = 0$ **and** $a = 2$

D. $a = 2$

E. $a = 0$ **and** $a = 1$

14. Let A and B be 3×3 matrices with $\det A = 3$ and $\det B = 2$. Then $\det(A$ adj $BA^TB^{-1})$ is

A. 9

B. 18

C. 36

D. 1/2

E. 1

15. Let A be a $n \times n$ invertible matrix. Which of the following statements are NOT true:

- **(i).** A^T **is nonsingular.**
- **(ii).** $A\mathrm{x} = \mathbf{0}$ **has only trivial solution.**
- **(iii).** A **is row equivalent to a matrix** B **that has a row of zeros.**
- **(iv).** $A\mathrm{x} = \mathrm{b}$ **has a unique solution for every** $n \times 1$ **matrix b.**
- **(v).** $|A| \neq 0$.

A. (i) and (iii).

B. (ii) and (v).

C. (iii) only.

D. (iii) and (iv).

E. (v) only.

16. Given $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 3$, **then** $\begin{vmatrix} -x & -y & -z \\ 3p+a & 3q+b & 3r+c \\ 2p & 2q & 2r \end{vmatrix} =$

A. -6

B. 6

C. 18

D. -18

E. -3

17. Find adj(A) if $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & 3 \\ 2 & 1 & 5 \end{bmatrix}$

A. $\begin{bmatrix} -8 & -6 & 10 \\ 6 & -3 & -3 \\ 2 & 3 & -1 \end{bmatrix}$

B. $\begin{bmatrix} -8 & 6 & 2 \\ -6 & -3 & -3 \\ 10 & -3 & -1 \end{bmatrix}$

C. $\begin{bmatrix} -8 & 6 & 2 \\ -6 & 3 & 3 \\ 10 & -3 & -1 \end{bmatrix}$

D. $\begin{bmatrix} -8 & 6 & 10 \\ 6 & -3 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

E. $\begin{bmatrix} -8 & 6 & 2 \\ -6 & -3 & 3 \\ -10 & -3 & -1 \end{bmatrix}$

18. For $n \times n$ **matrices with real entires** A **and** B**, decide which statements are ALWAYS true.**

I. $\det(A + B) = \det(A) + \det(B)$

II. If $\det(A) \neq 0$ **then** $\det(adj(A)) = \dfrac{1}{\det(A)}$

III. For any real number c**,** $\det(cA) = c\det(A)$**.**

A. None of the statements are true

B. I only

C. II only

D. III only

E. II and III only

19. The matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **is nonsingular if and only if**

A. $a \neq 0,\ b \neq 0$

B. $ad - bc \neq 0$

C. $a = b = c = d = 1$

D. $ad - bc = 0$

E. $a = b = c = d$

20. If A **is an** $n \times n$ **singular matrix, then** $A(\textbf{adj}A) =$
(Note that $O_{n\times n}$ **is the** $n \times n$ **matrix with all zeros while 0 means just the number 0).**

A. $O_{n\times n}$

B. I_n

C. 0

D. A^{-1}

E. 1

21. **Consider the solution** $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ **to the system** $\begin{bmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. **Decide which equality below is true for all** $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ **in** $\mathbf{R}^3$.

A. $x_2 = \begin{vmatrix} 11 & a & 13 \\ 4 & b & 8 \\ 3 & c & 9 \end{vmatrix}$

B. $x_1 = \begin{vmatrix} a & b & c \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}$

C. $x_3 = -\dfrac{\begin{vmatrix} 4 & 12 & b \\ 11 & 7 & a \\ 3 & 7 & c \end{vmatrix}}{\begin{vmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}}$

D. $x_2 = \dfrac{b}{\begin{vmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}}$

E. $x_1 = \dfrac{\begin{vmatrix} a & 4 & 3 \\ b & 12 & 7 \\ c & 8 & 9 \end{vmatrix}}{\begin{vmatrix} 11 & 7 & 13 \\ 4 & 12 & 8 \\ 3 & 7 & 9 \end{vmatrix}}$

22. **The heads of three vectors OA, OB and OC (where O is the origin) are:** $A = (1, 2, 5)$, $B = (5, 0, -1)$ **and** $C = (-2, 2, 1)$ **and they form a triangle, which side(s) of the triangle ABC is (are) the longest.**

A. **AB**

B. **AC**

C. **BC**

D. **AB and AC**

E. **AB and BC**

23. Which of the following subset of R^2 with the usual operations of vector addition and scalar multiplication are subspaces?

I. W_1 **is the set of all vectors of the form** $\begin{bmatrix} x \\ y \end{bmatrix}$**, where** $x \geq 0$.

II. W_2 **is the set of all vectors of the form** $\begin{bmatrix} x \\ y \end{bmatrix}$**, where** $x \geq 0$ **and** $y \geq 0$**.**

III. W_3 **is the set of all vectors of the form** $\begin{bmatrix} x \\ y \end{bmatrix}$**, where** $x = 0$.

A. W_1 **only;**

B. W_2 **only;**

C. W_3 **only;**

D. W_1 **and** W_2**;**

E. W_2 **and** W_3**.**

24. Which of the following subsets of the vector space M_{nn} **are subspace?**

I. The set of all $n \times n$ **symmetric matrices;**

II. The set of all $n \times n$ **diagonal matrices;**

III. The set of all $n \times n$ **singular matrices;**

A. I **only;**

B. II **only;**

C. III **only;**

D. I **and** II**;**

E. II **and** III**.**